CS143: B+Tree

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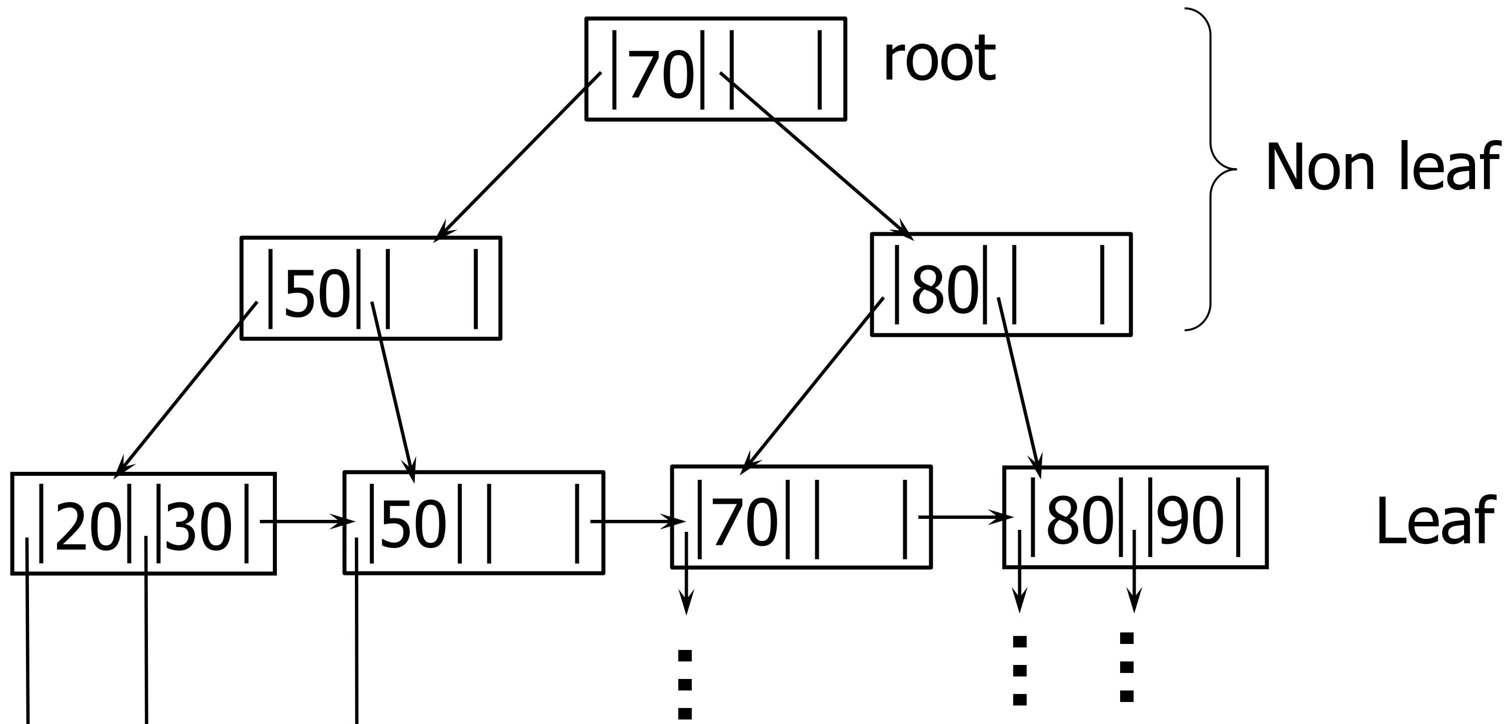
B+Tree

- Most popular index structure in RDBMS
- Advantage
  - Suitable for dynamic updates
  - Balanced
  - Minimum space usage guarantee
- Disadvantage
  - Non-sequential index blocks

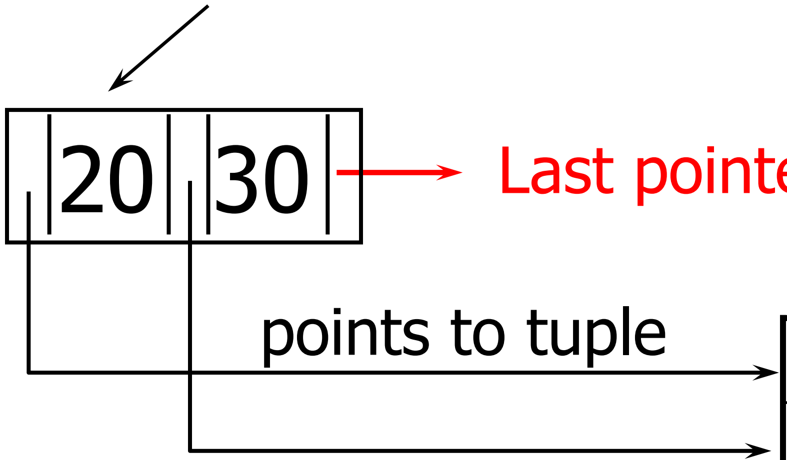
B+Tree (n=3)



20	Susan	2.7
30	James	3.6
50	Peter	1.8
…	…	…

- n: # of pointer spaces in a node
- Balanced: All leaf nodes are at the same level

Leaf Node (n=3)

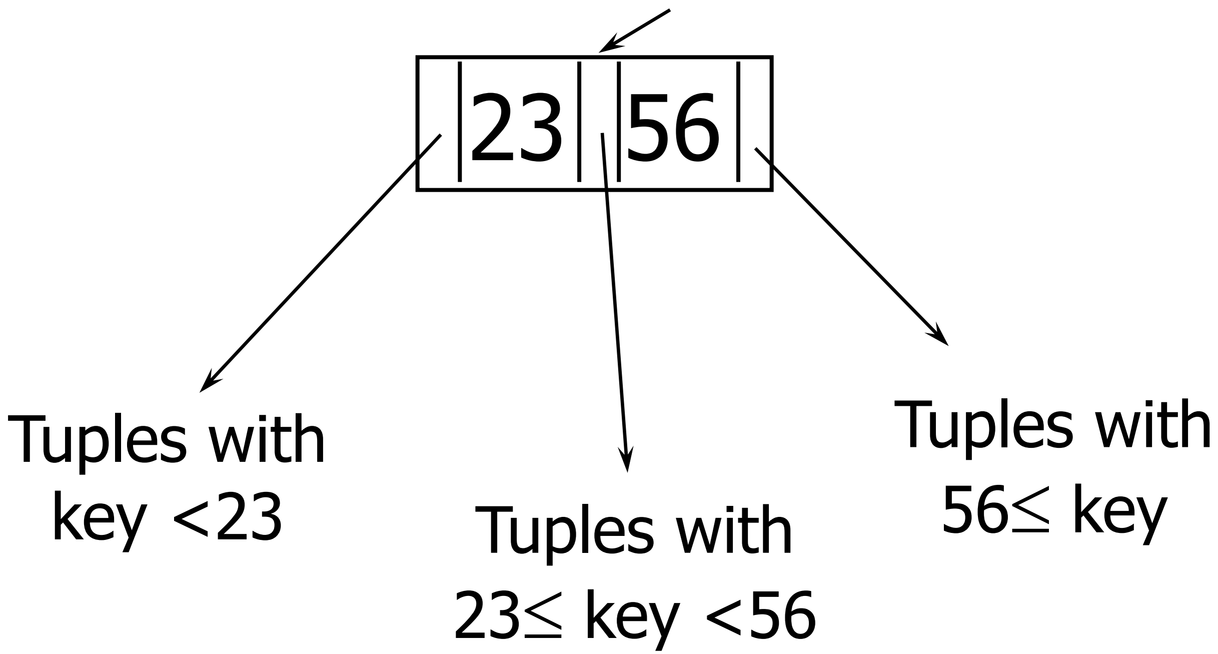


Last pointer: to the next leaf node

20	Susan	2.7
30	James	3.6
50	Peter	1.8
…	…	…

- All pointers (except the last one) point to tuples
- At least half of the pointer spaces are used.
  (more precisely, $\lceil (n+1)/2 \rceil$ pointers)

Non-leaf Node (n=3)

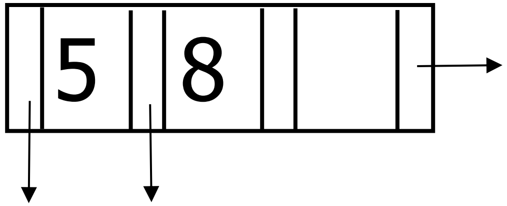
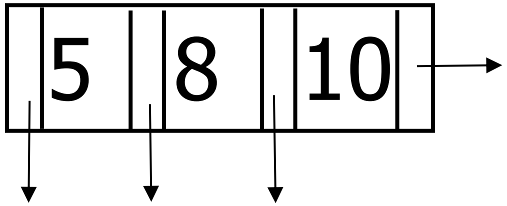
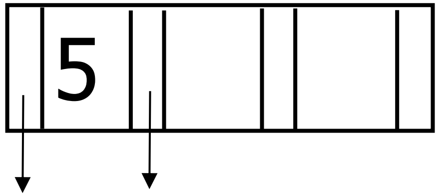
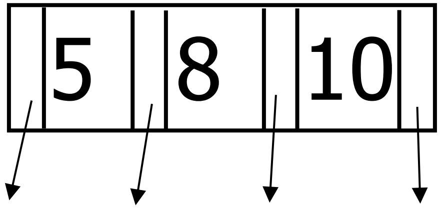


- Points to the nodes one-level below
  - No direct pointers to tuples
- At least half of the pointer spaces used (precisely, $\lceil n/2 \rceil$)
  - except root, where at least 2 pointer spaces used

Space Usage Guarantee

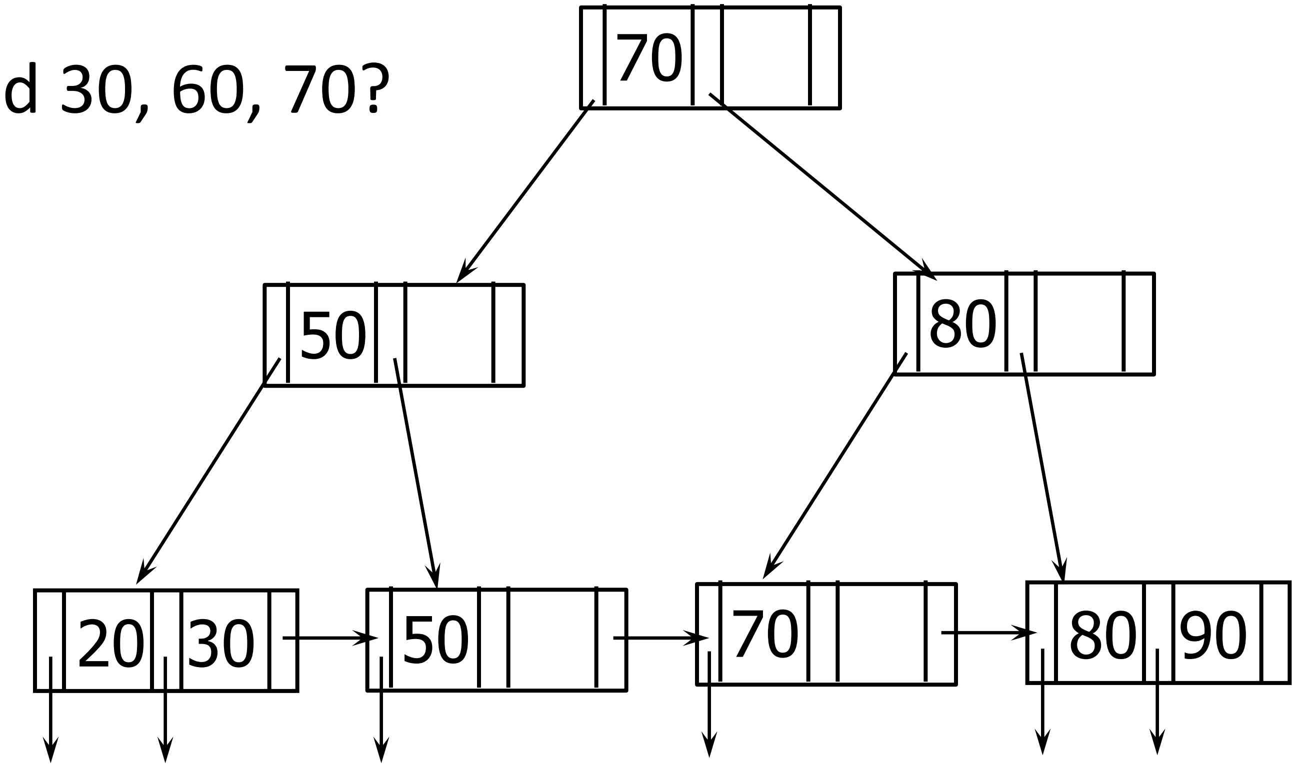
$\lceil \frac{5}{2} \rceil = \lceil 2.5 \rceil = 3$ $\quad \lceil \frac{5}{2} \rceil - 1 = 2$

- B+Tree nodes have at least
  - Leaf (non-root): $\lceil (n+1)/2 \rceil$ pointers, $\lceil (n+1)/2 \rceil - 1$ keys
  - Non-leaf (non-root): $\lceil n/2 \rceil$ pointers, $\lceil n/2 \rceil - 1$ keys
  - Root: 2 pointers, 1 key

n=4	Minimum	Full
Leaf		
Non-leaf		

Search on B+tree

- Find 30, 60, 70?



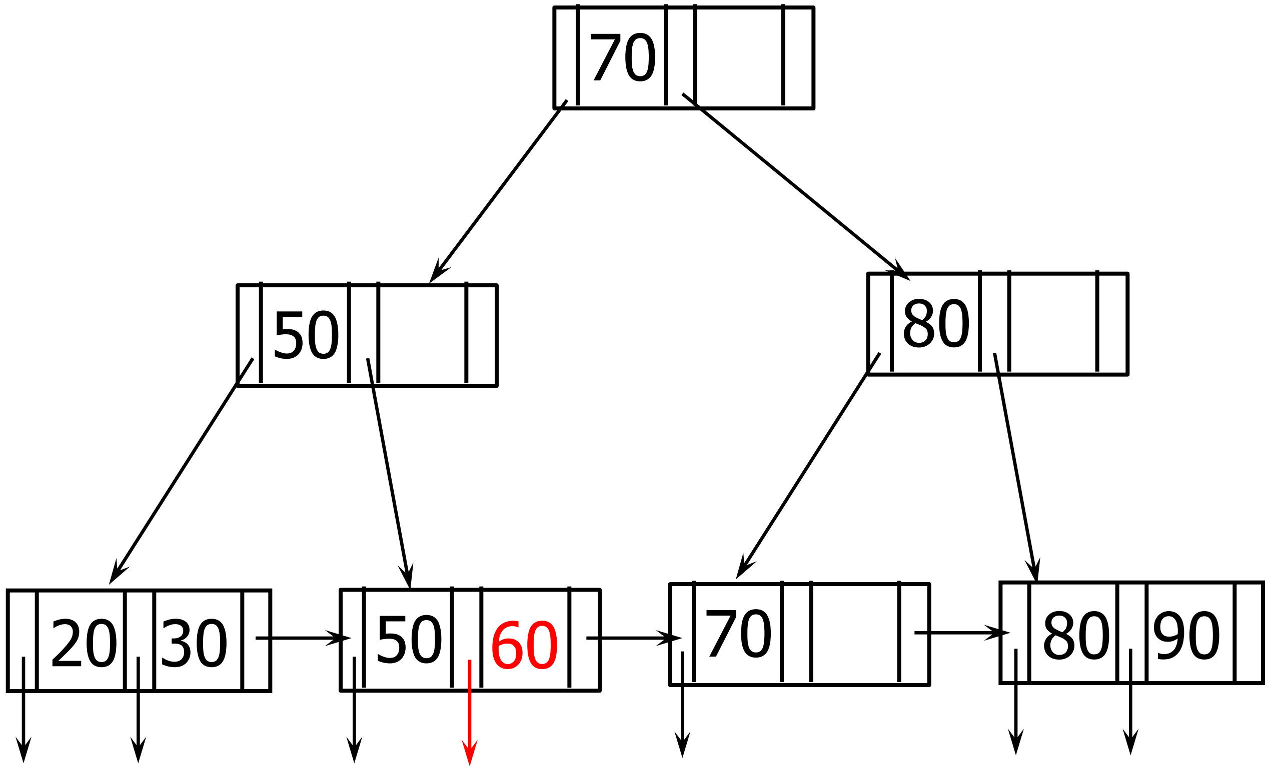
- Find a greater key and follow the link on the left
  (Algorithm: Figure 14.11 on textbook)

B+Tree Insertion

1. no overflow
2. leaf overflow
3. non-leaf overflow
4. new root

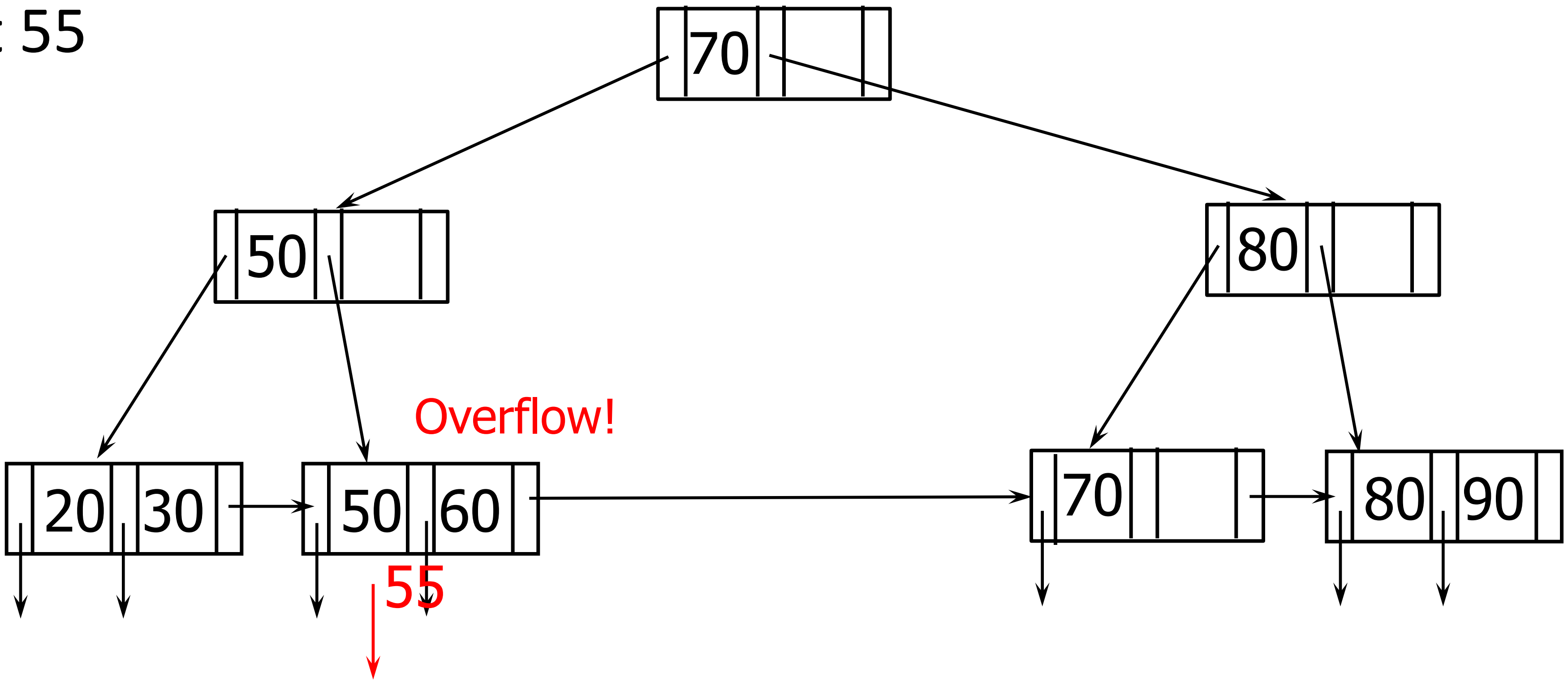
1. No Overflow

- Insert 60



2. Leaf Overflow

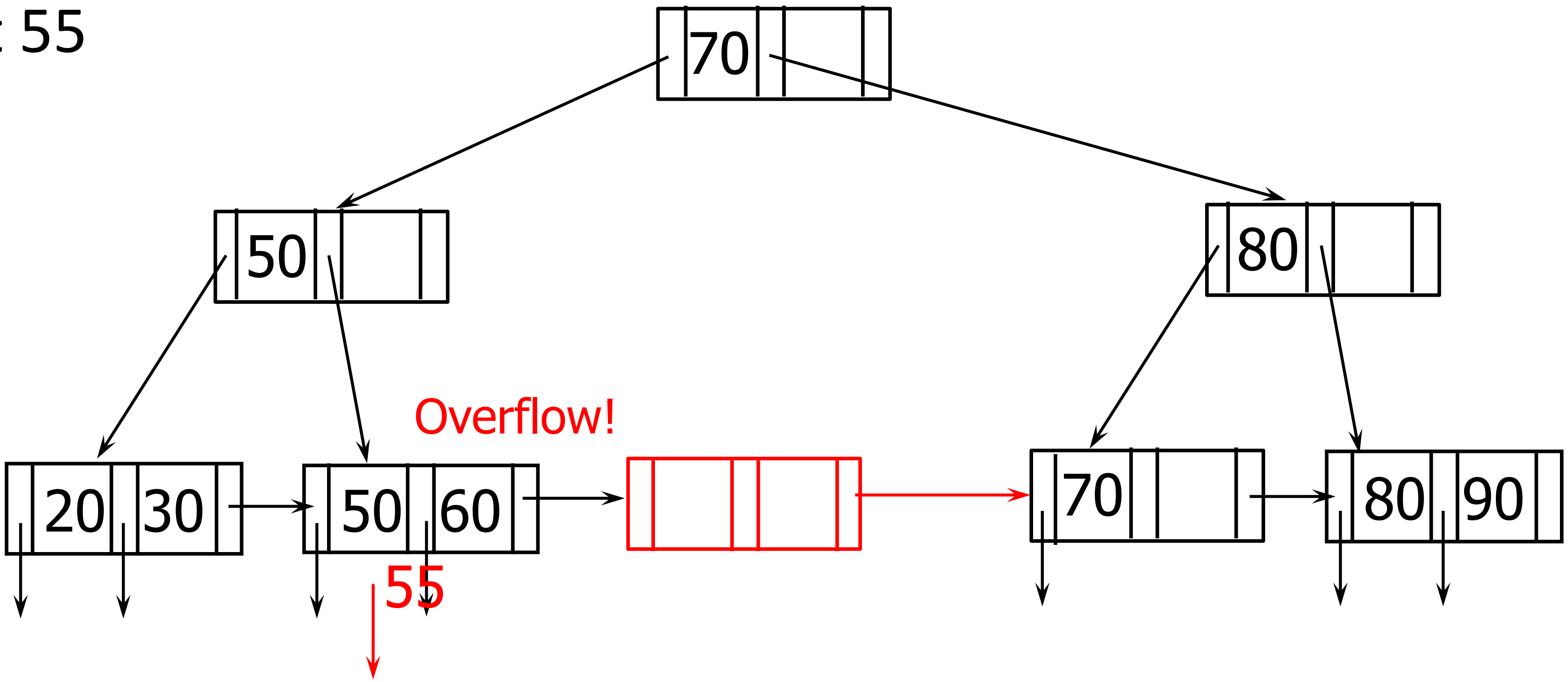
- Insert 55



- No space to store 55

2. Leaf Overflow

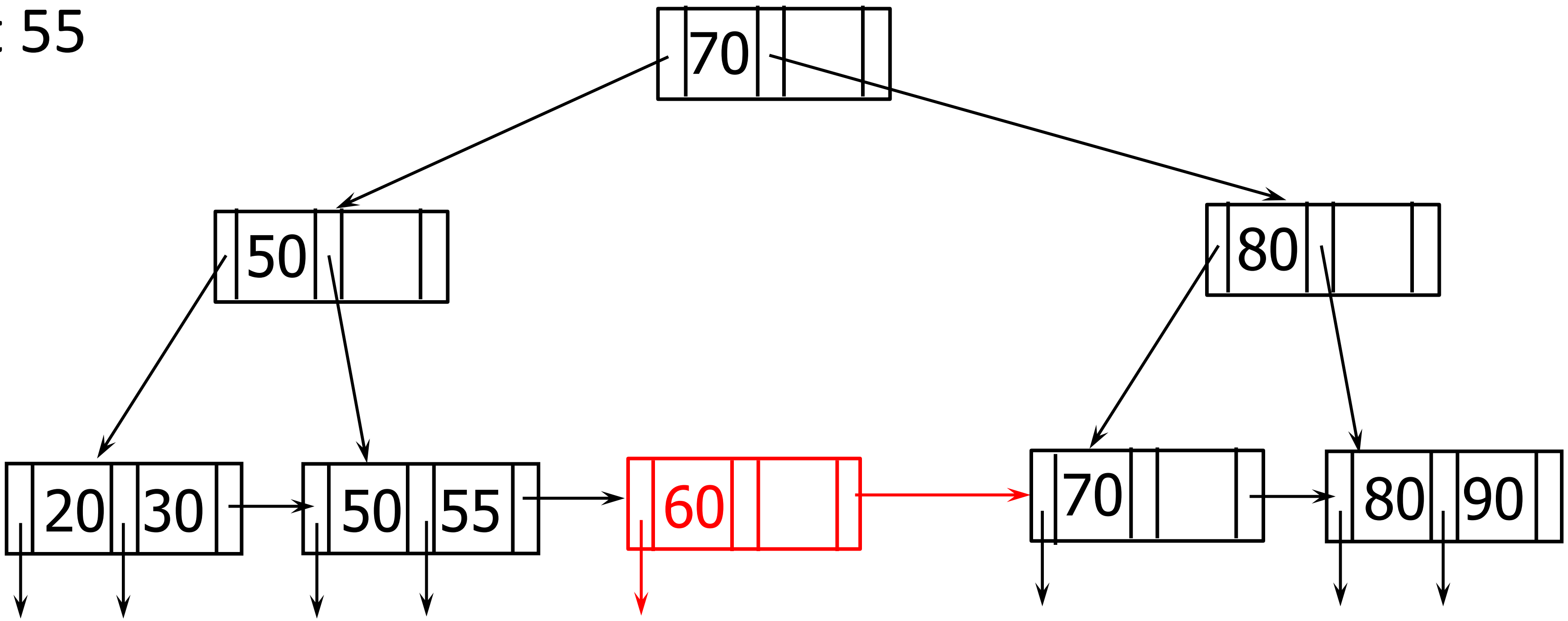
- Insert 55



- Split the leaf into two. Put the keys half and half

2. Leaf Overflow

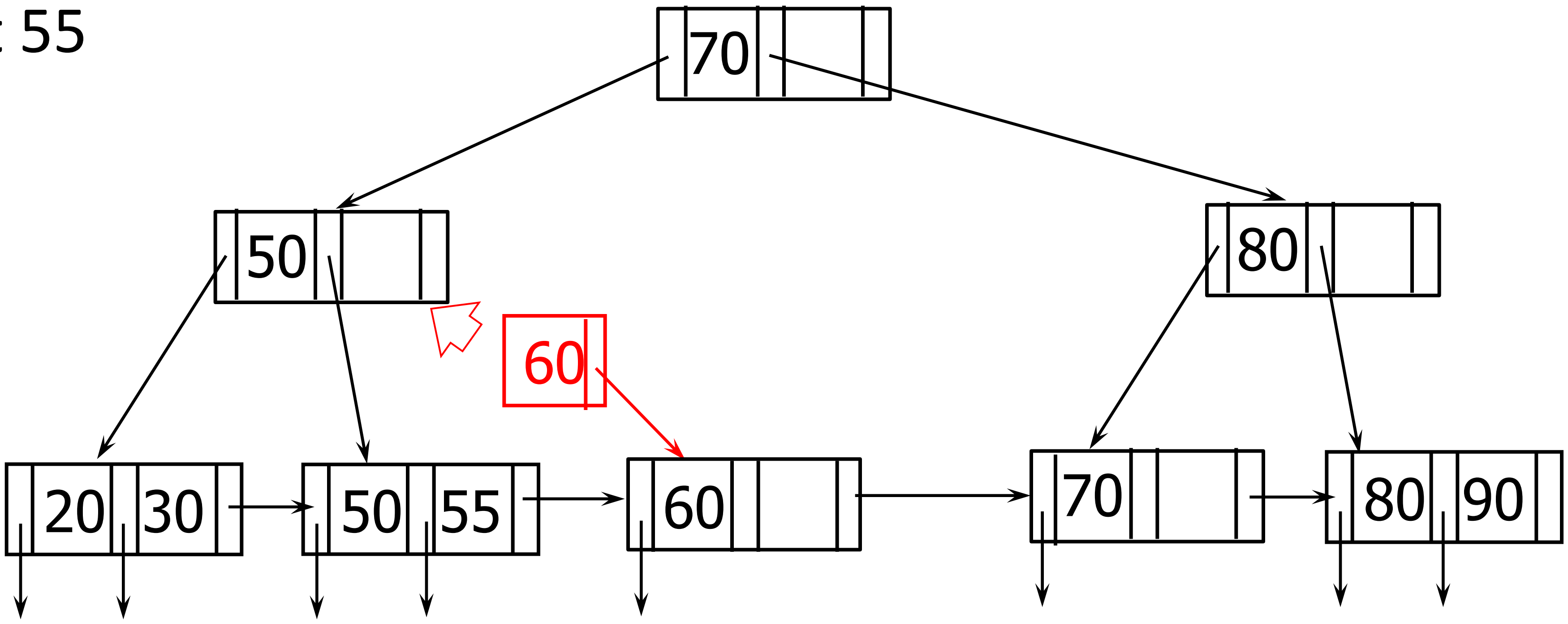
- Insert 55



- Split the leaf into two. Put the keys half and half

2. Leaf Overflow

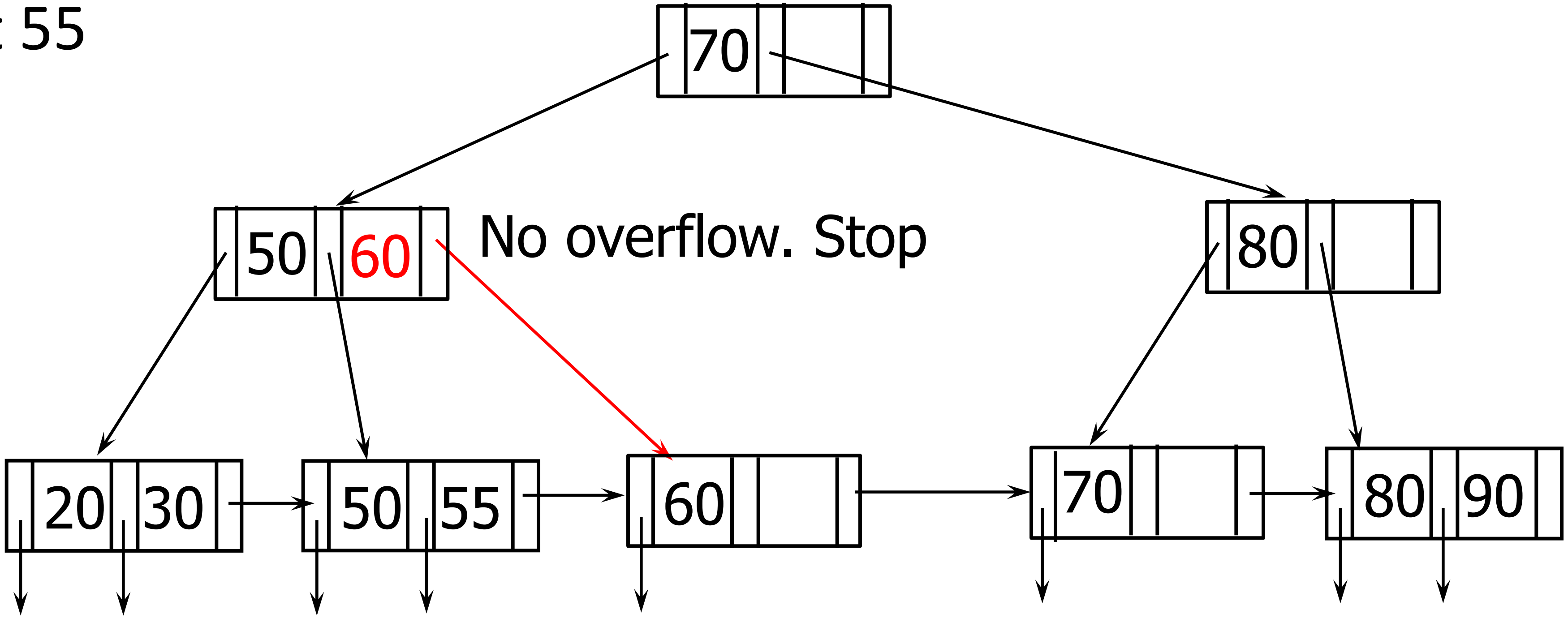
- Insert 55



- *<u>Copy</u>* the first key of the new node to parent

2. Leaf Overflow

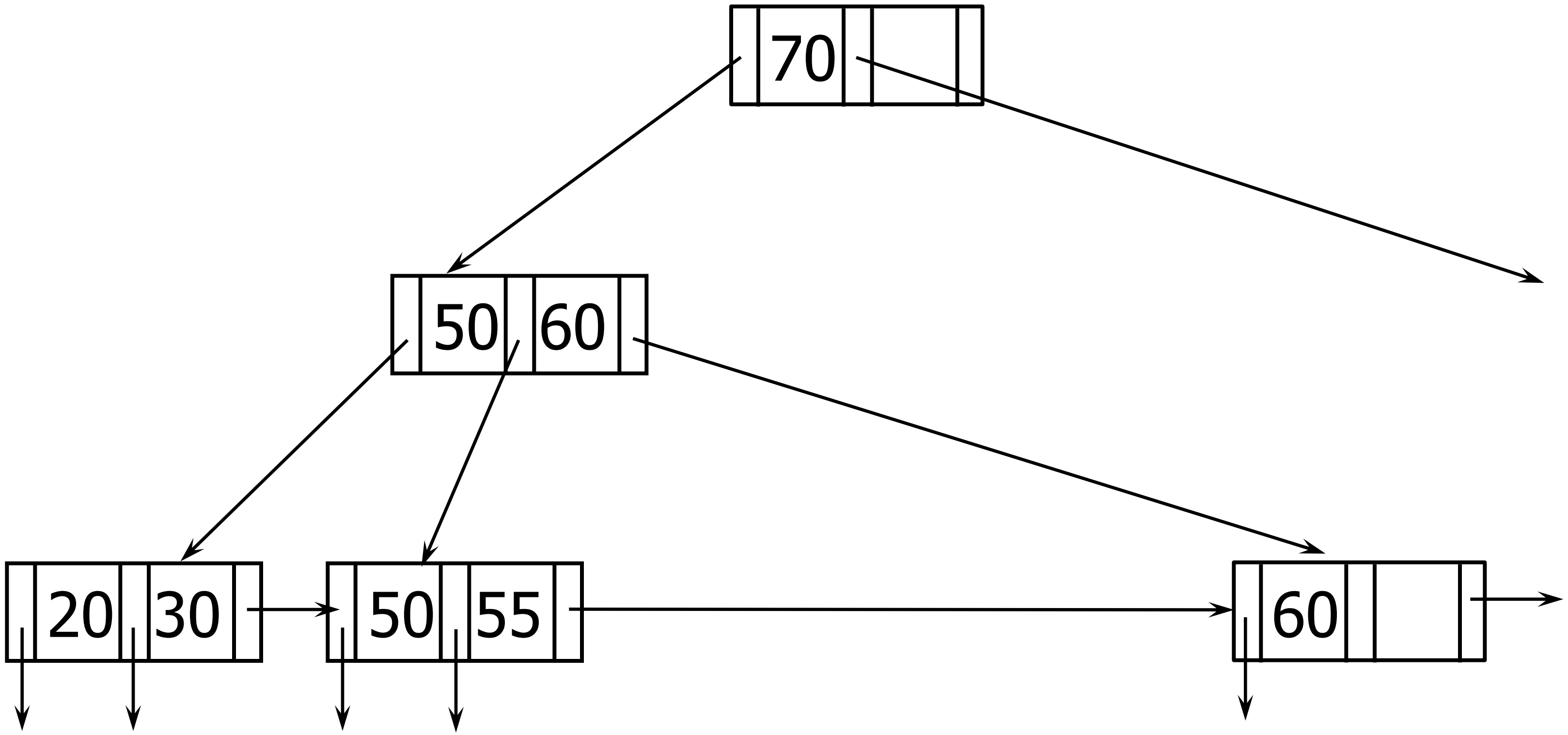
- Insert 55



- Q: After split, leaf nodes always half full?

3. Non-leaf Overflow

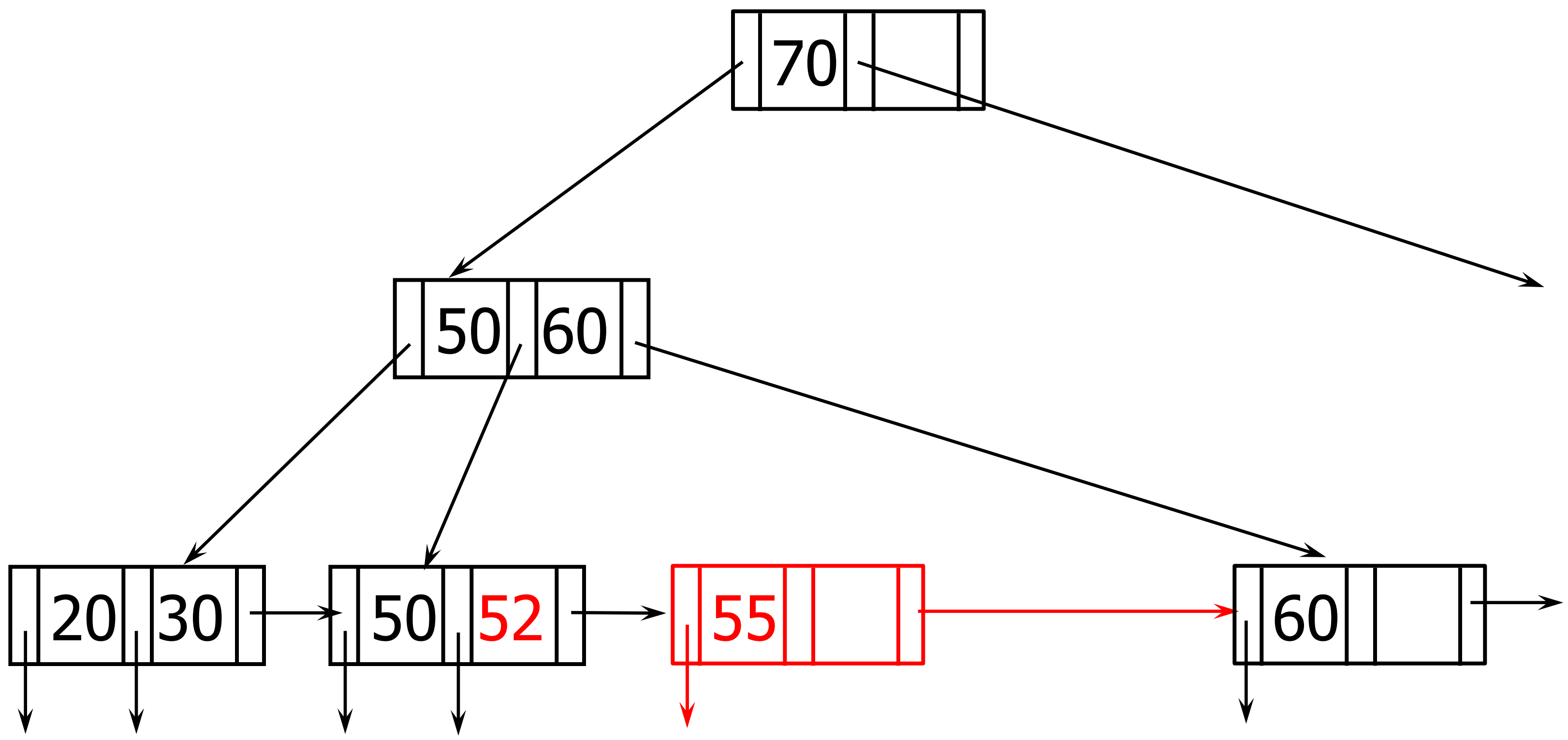
- Insert 52



Leaf overflow. Split and copy the first key of the new node

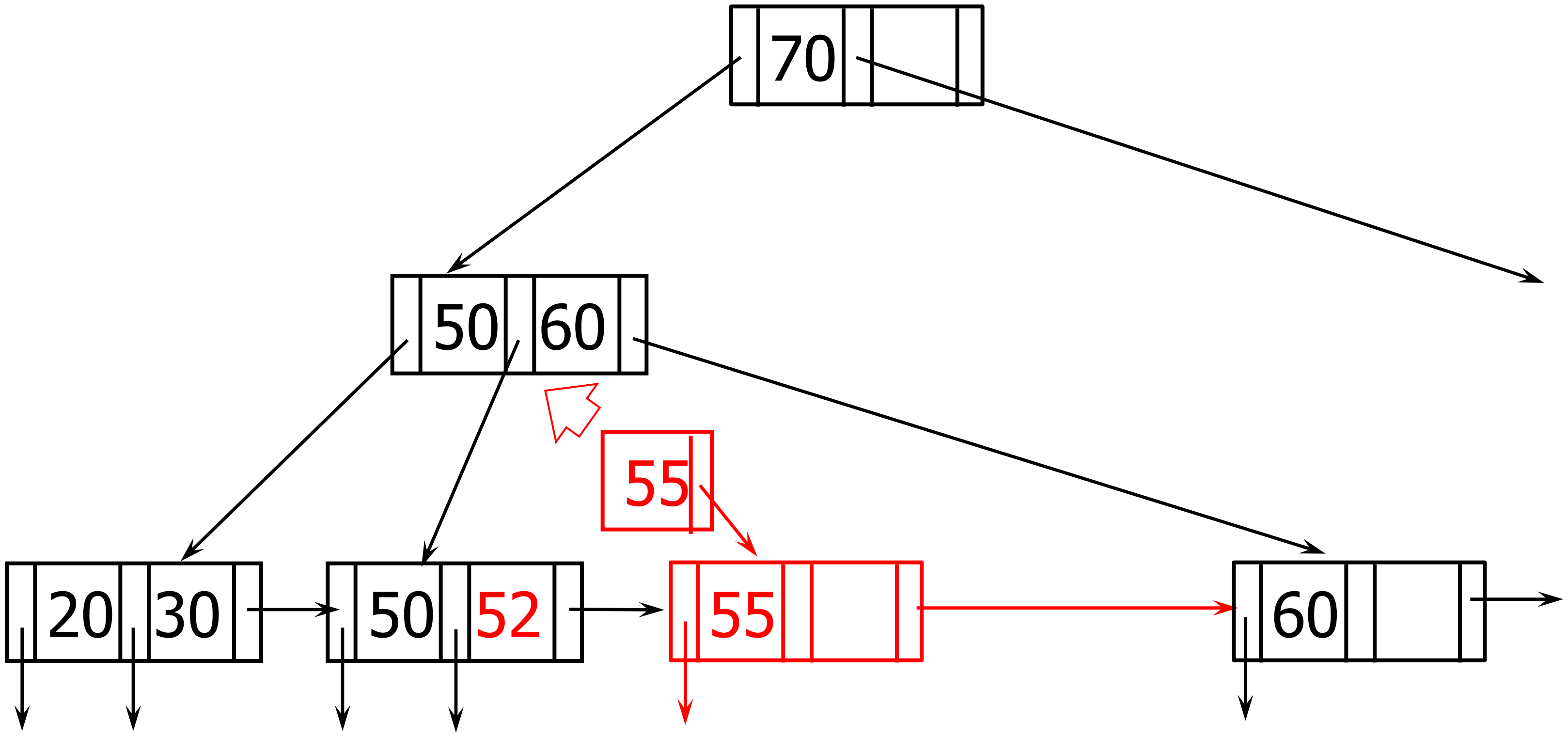
3. Non-leaf Overflow

- Insert 52



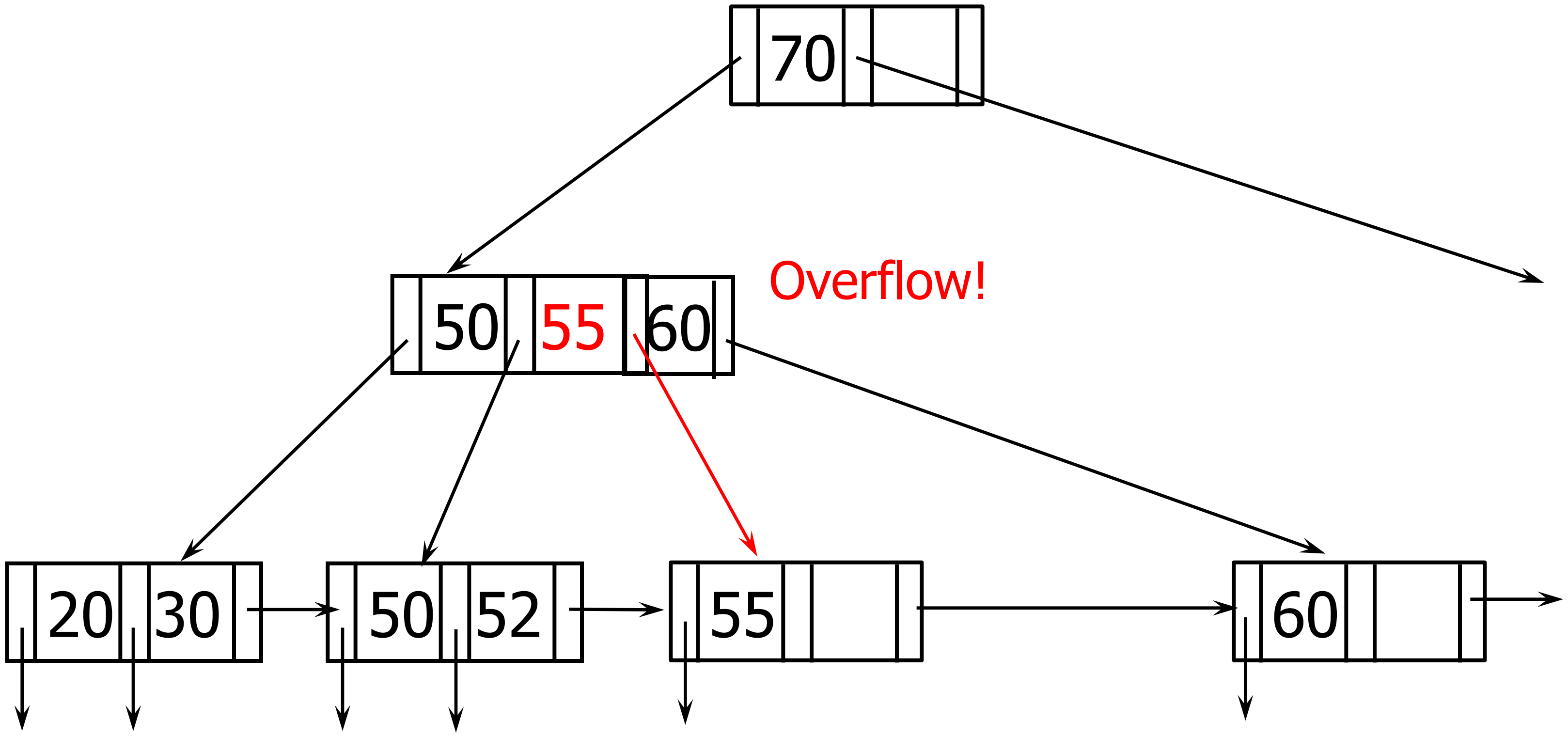
3. Non-leaf Overflow

- Insert 52



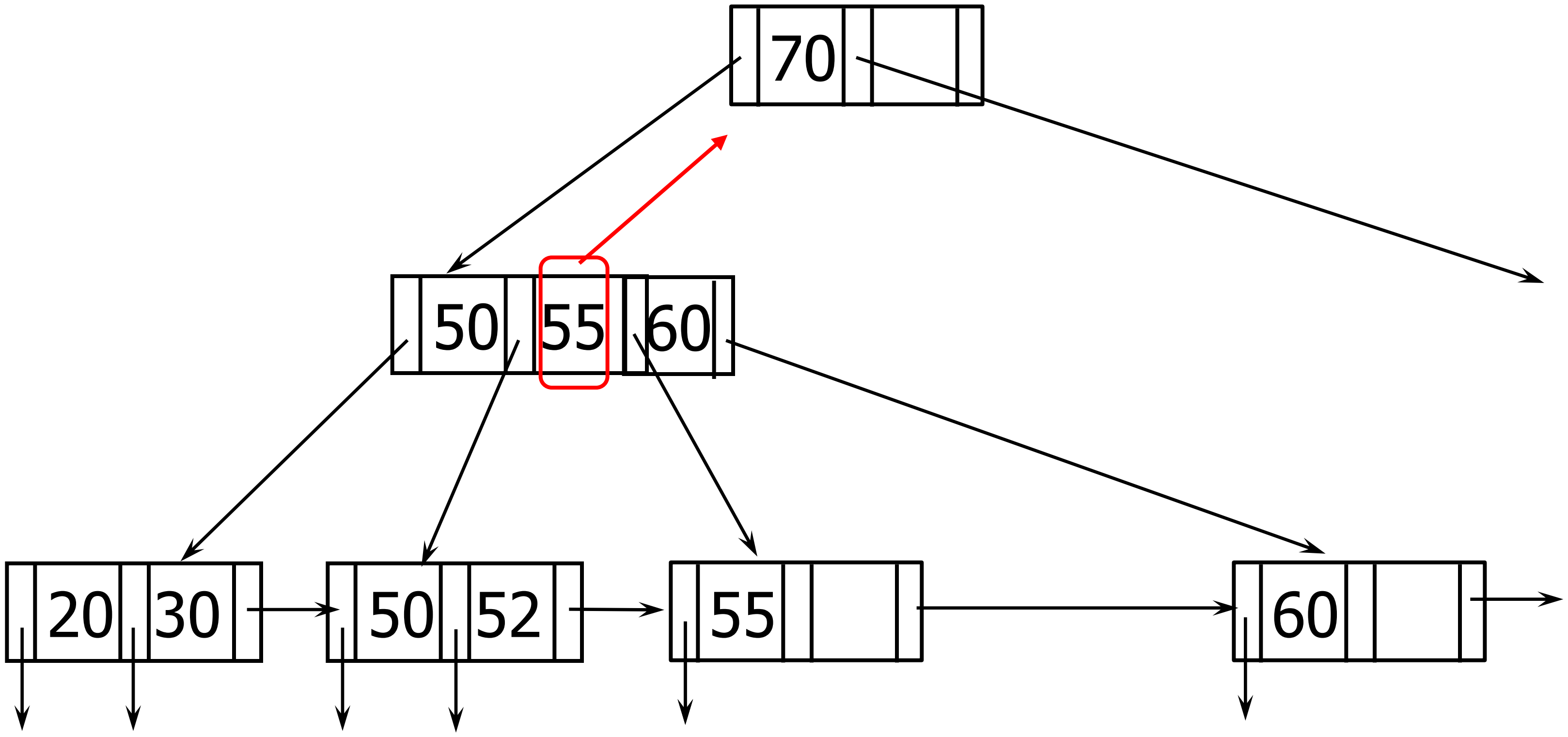
3. Non-leaf Overflow

- Insert 52



3. Non-leaf Overflow

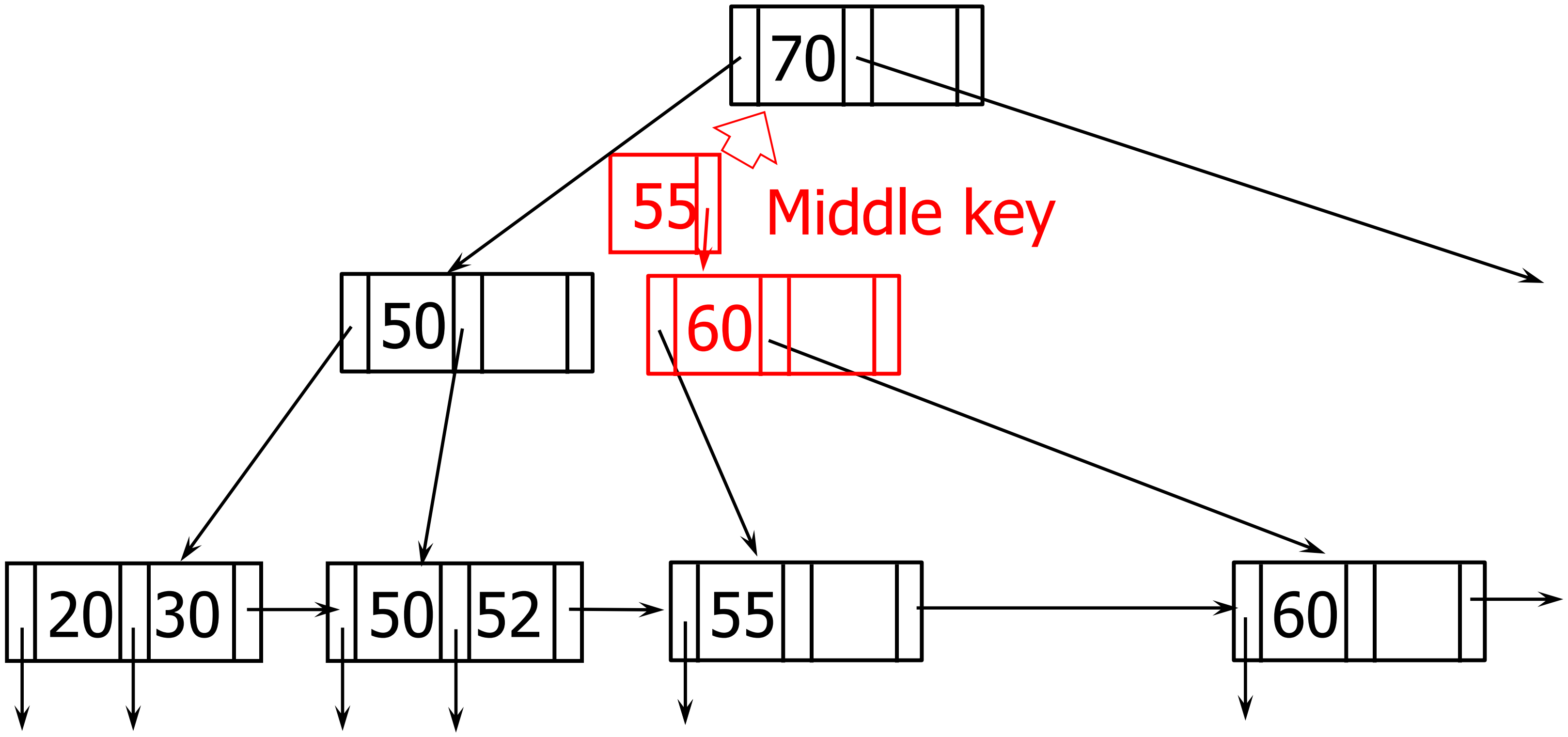
- Insert 52



Split the node into two. *Move* up the key in the middle.

3. Non-leaf Overflow

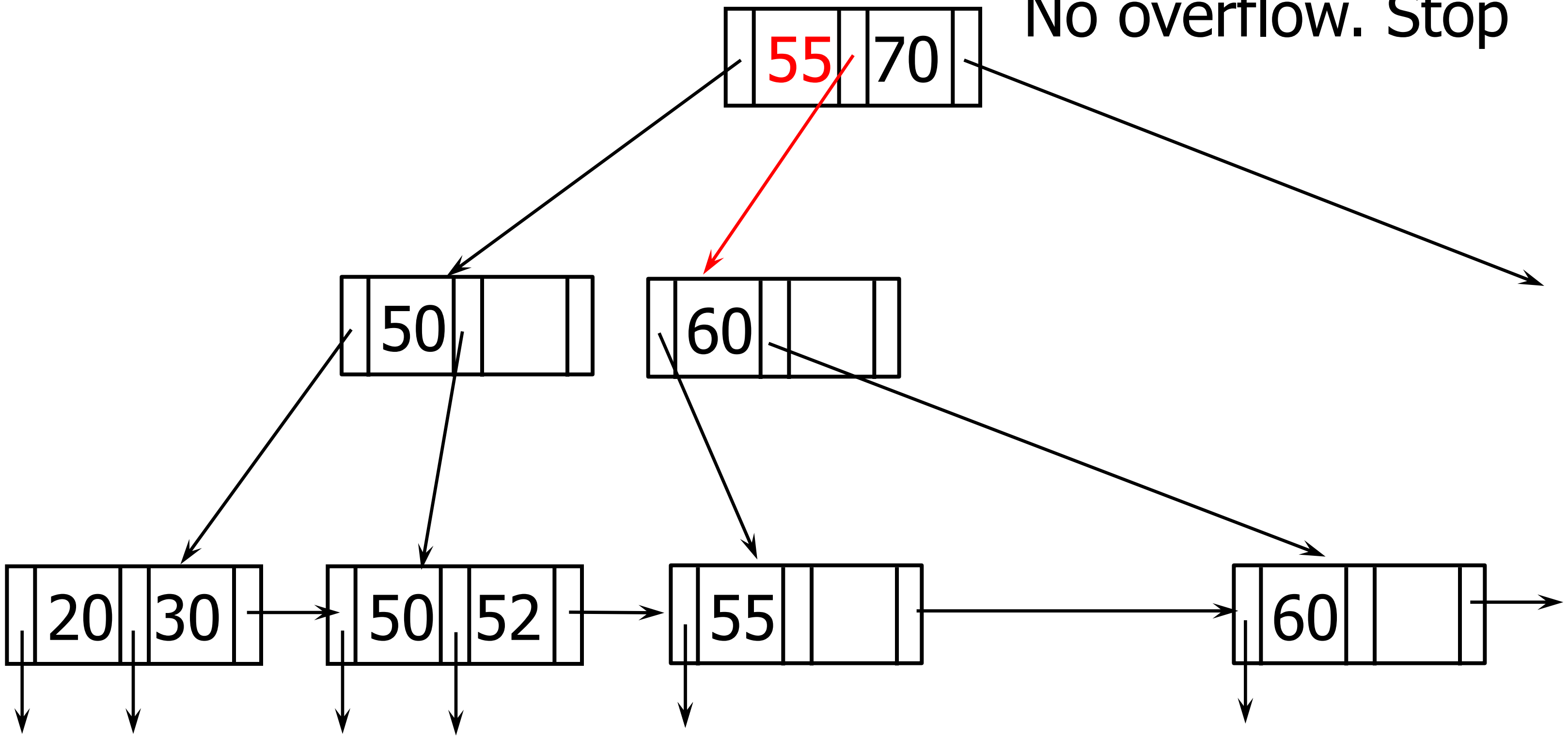
- Insert 52



3. Non-leaf Overflow

- Insert 52

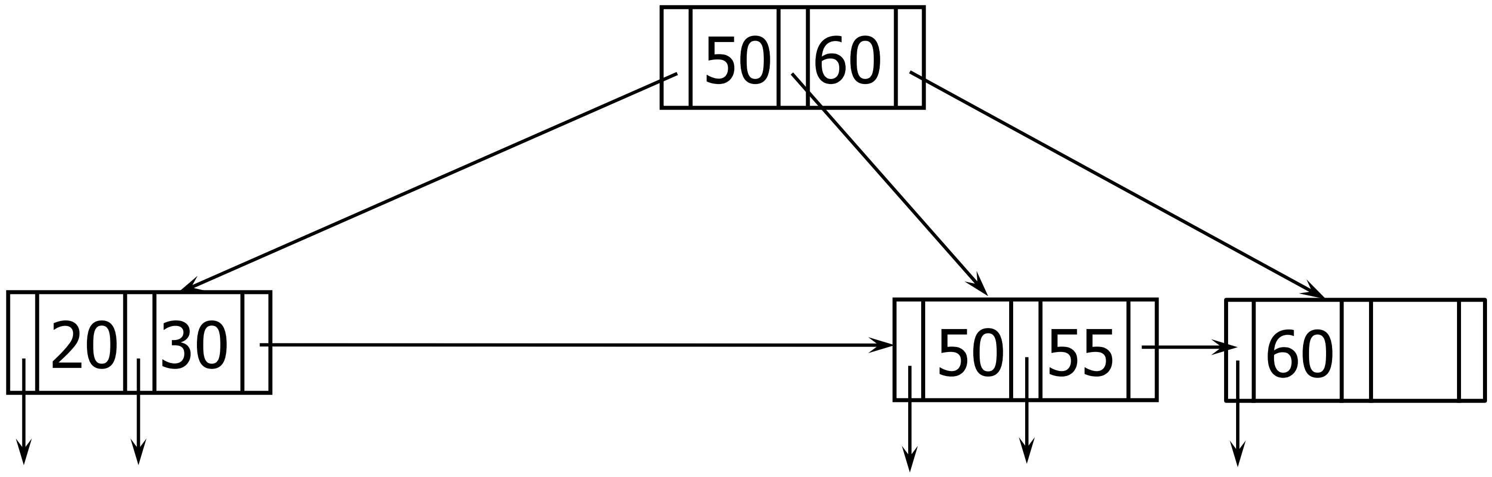
No overflow. Stop



Q: After split, non-leaf at least half full?

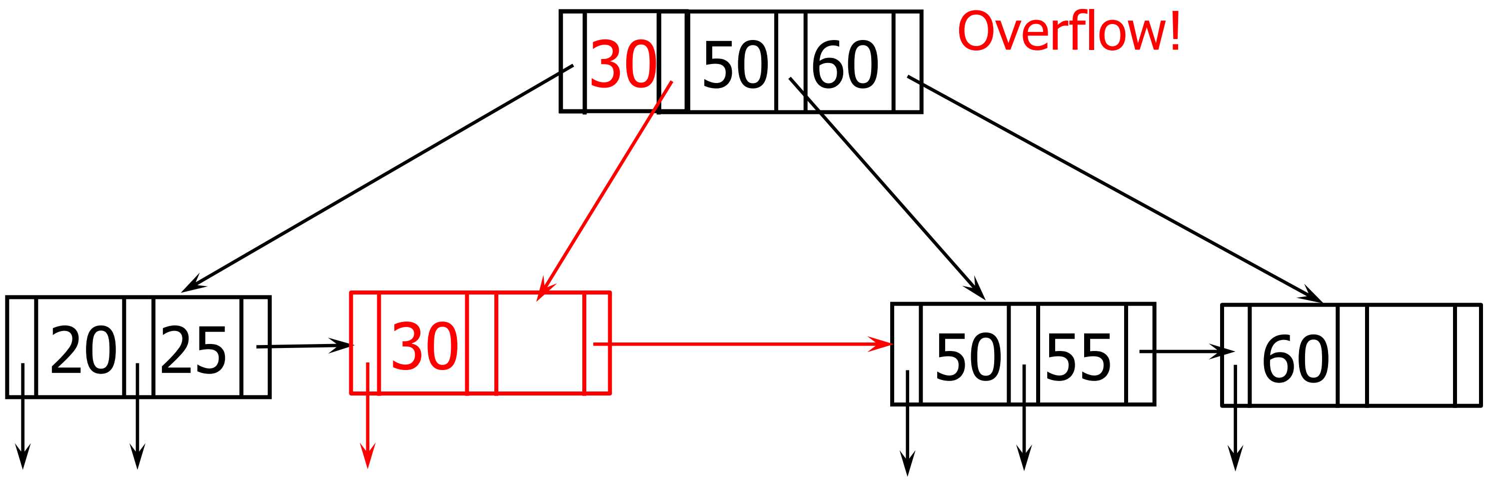
4. New Root

- Insert 25



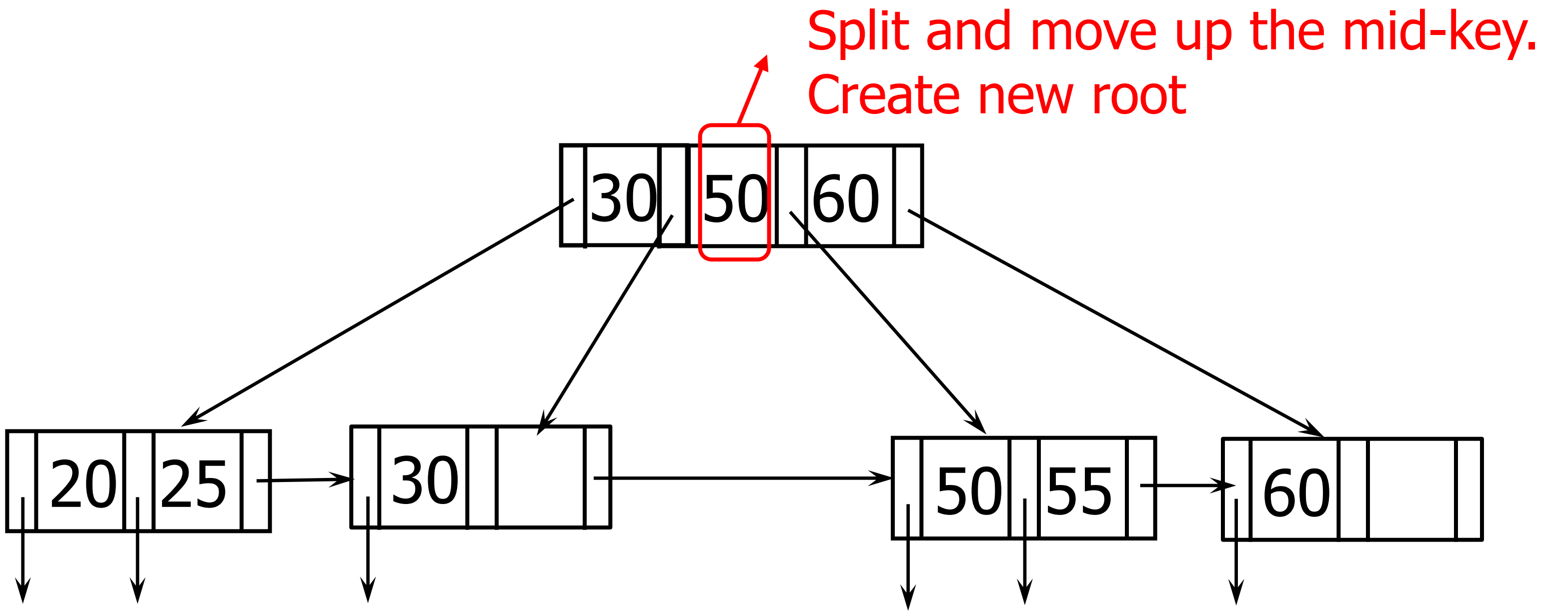
4. New Root

- Insert 25



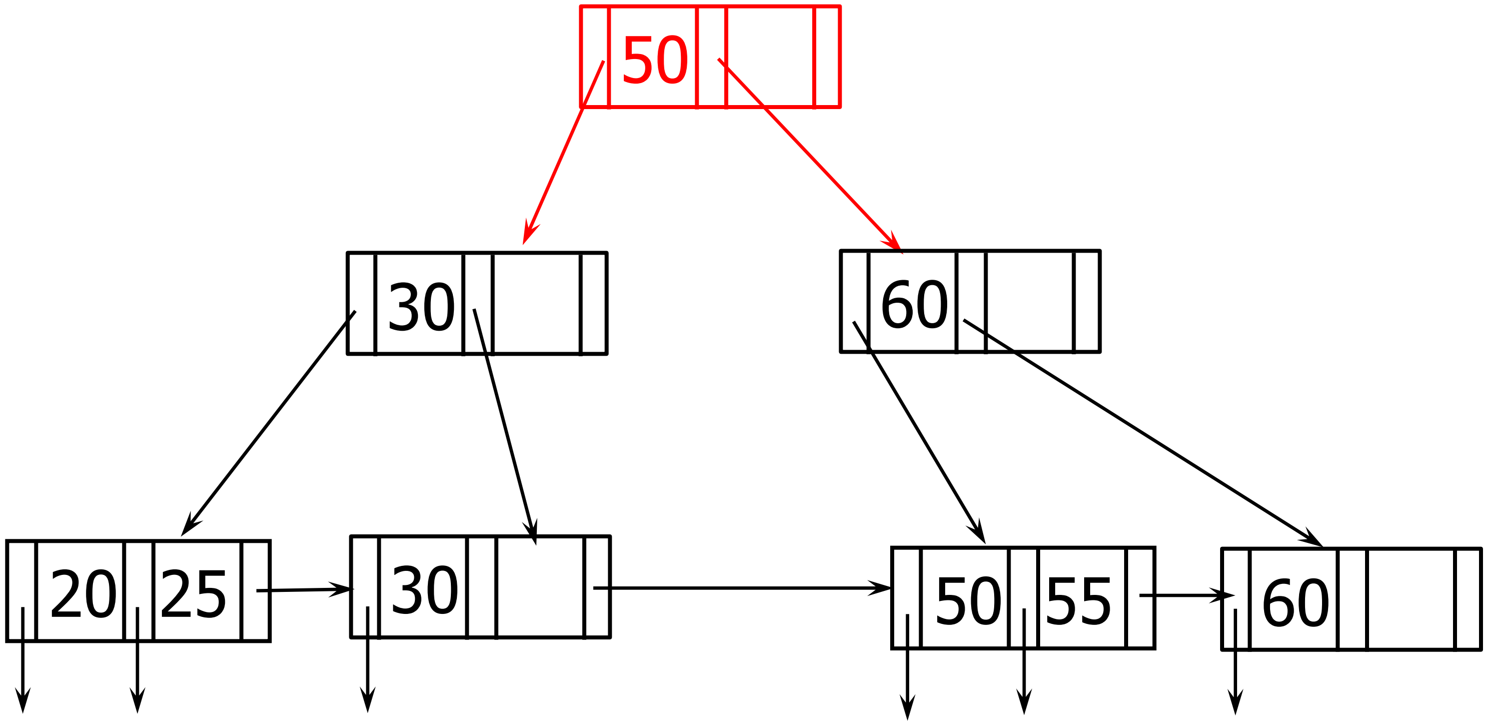
4. New Root

- Insert 25



4. New Root

- Insert 25
- Q: At least 2 pointers at root?



B+Tree Insertion

- Leaf node overflow
  - The first key of the new node is *copied* to the parent
- Non-leaf node overflow
  - The middle key is *moved* to the parent
- Detailed algorithm: Figure 14.17

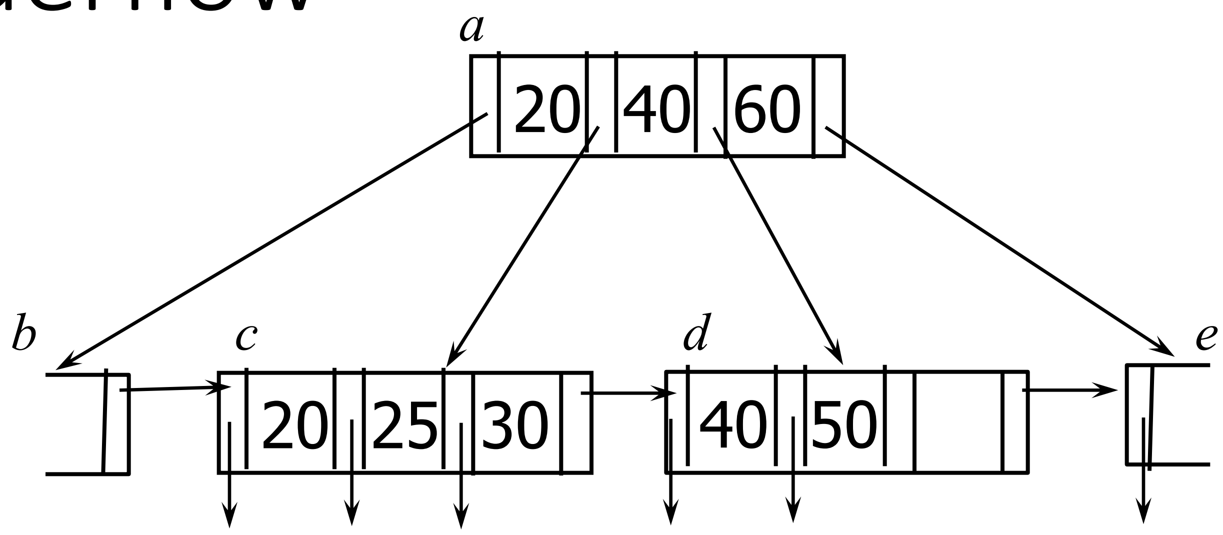
B+Tree Deletion

1. No underflow
2. Leaf underflow (coalesce with neighbor)
3. Leaf underflow (redistribute with neighbor)
4. Non-leaf underflow (coalesce with neighbor)
5. Non-leaf underflow (redistribute with neighbor)
6. Tree depth reduction

In the examples, n = 4

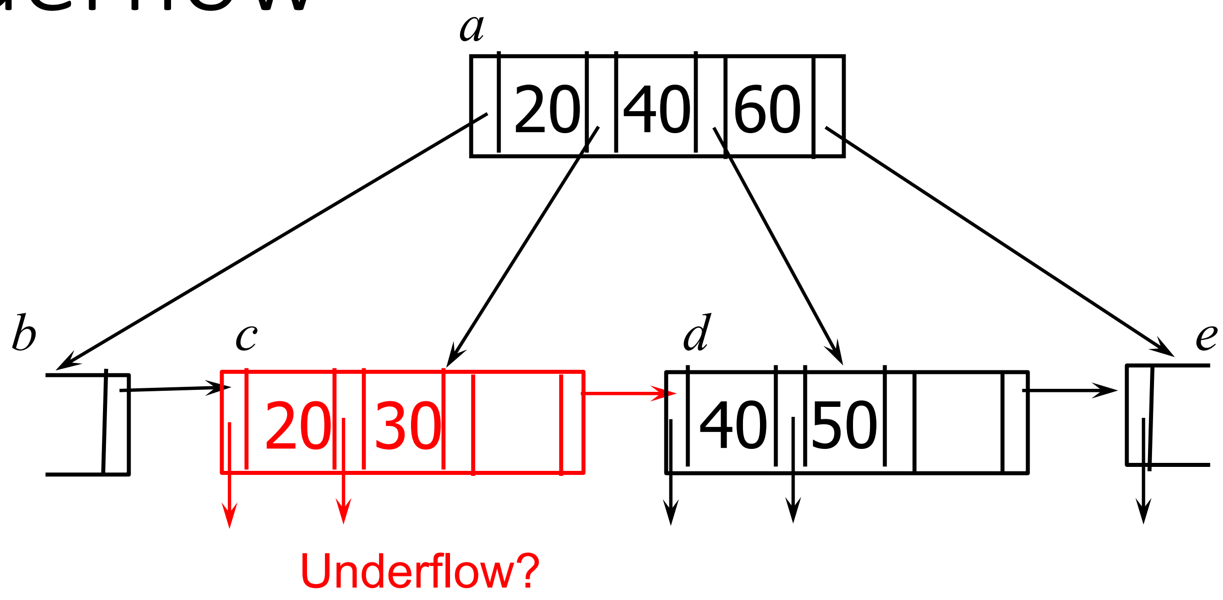
- Underflow for non-leaf when fewer than $\lceil n/2 \rceil$ = 2 pointers
- Underflow for leaf when fewer than $\lceil (n+1)/2 \rceil$ = 3 pointers
- Nodes are labeled as *a, b, c, d, …*

1. No Underflow



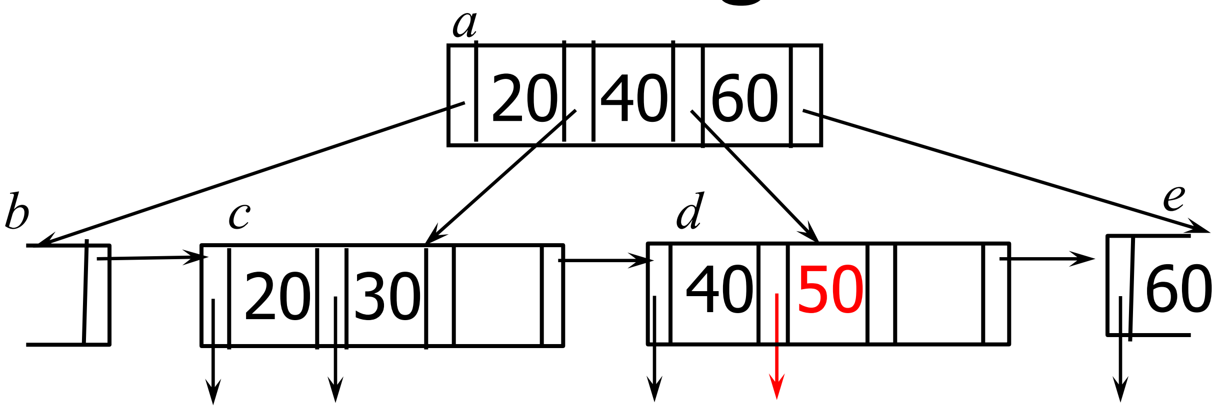
- Delete 25

1. No Underflow



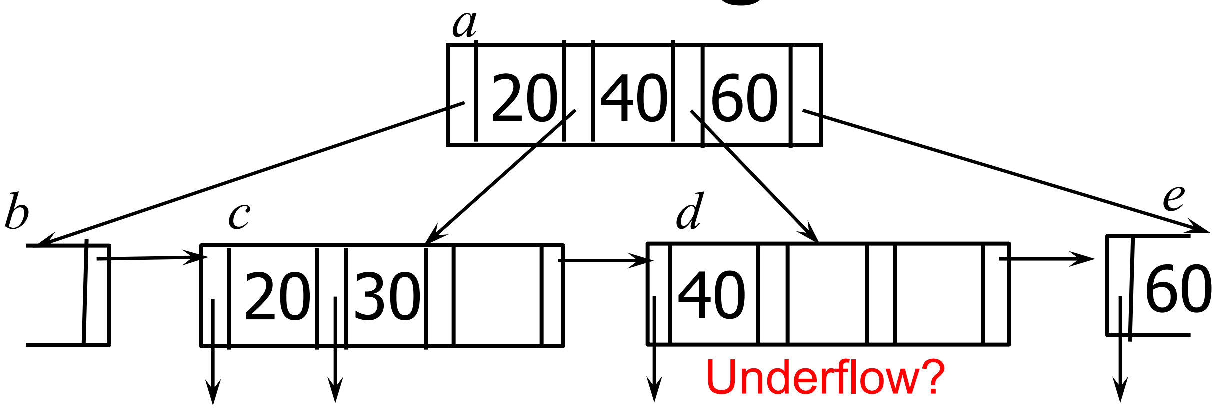
- Delete 25
  - Underflow? Min 3 ptrs. Currently 3 ptrs

2. Coalesce Leaf with Neighbor



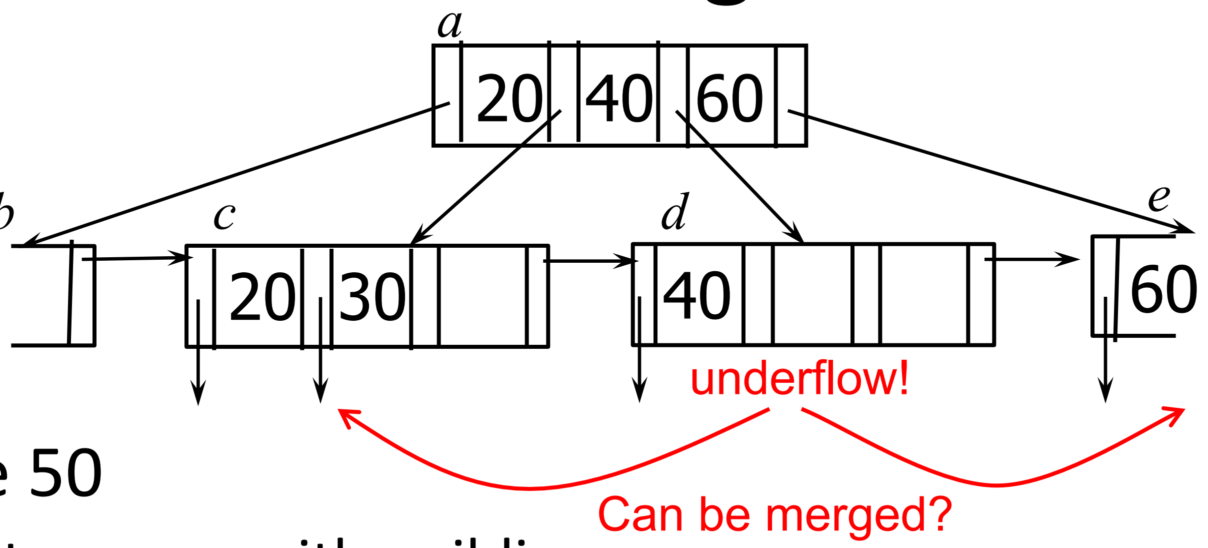
- Delete 50

2. Coalesce Leaf with Neighbor



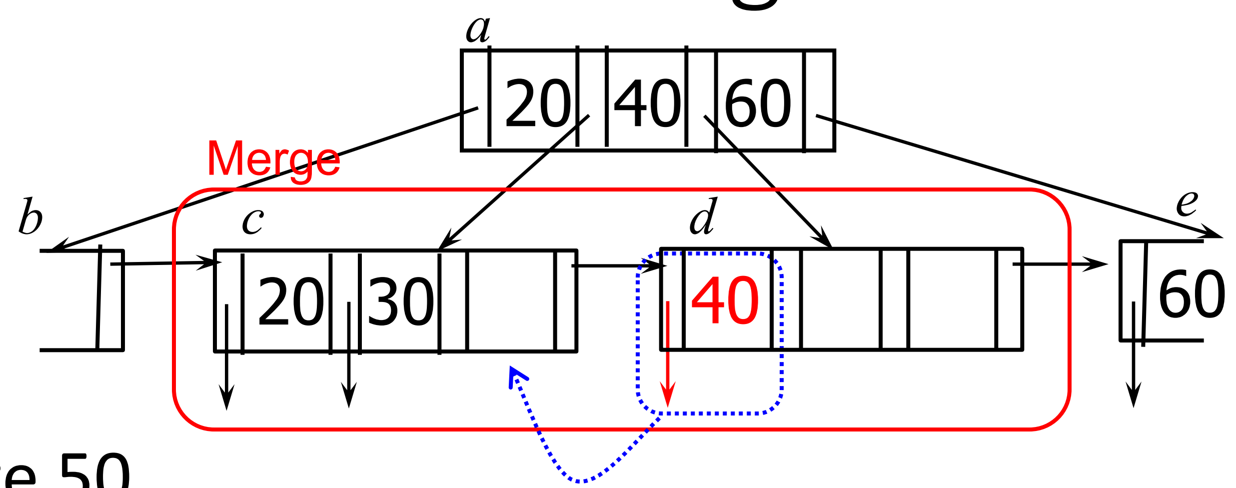
- Delete 50
  - Underflow? Min 3 ptrs, currently 2.

2. Coalesce Leaf with Neighbor



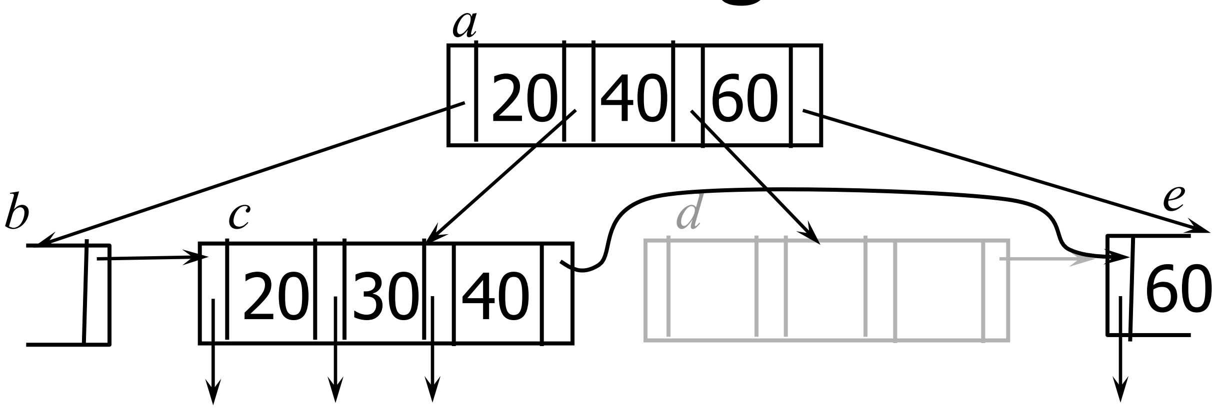
- Delete 50
  - Try to merge with a sibling

2. Coalesce Leaf with Neighbor



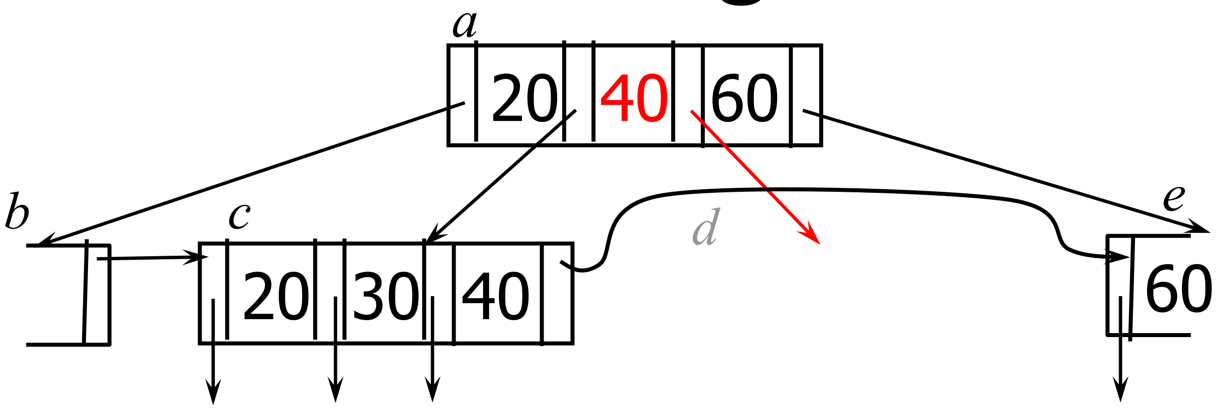
- Delete 50
  - Merge c and d. Move everything on the right to the left.

2. Coalesce Leaf with Neighbor



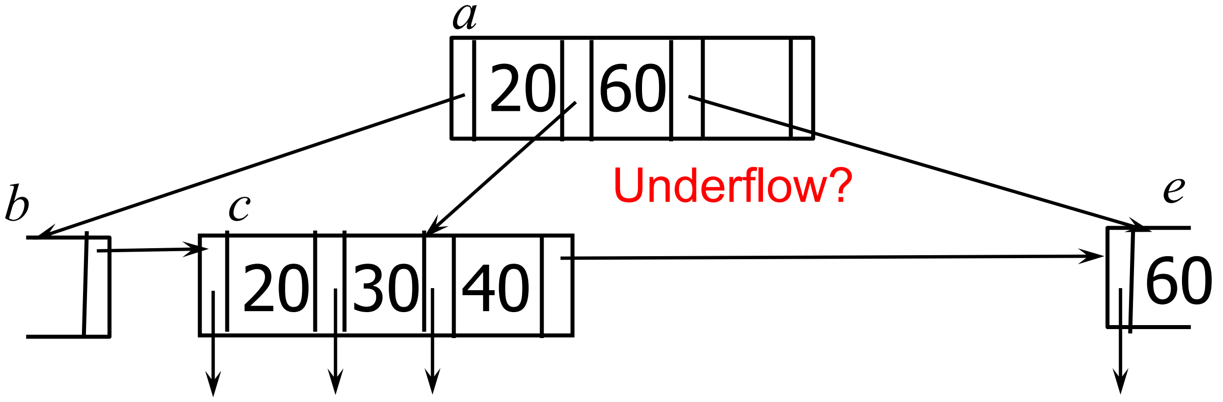
- Delete 50
  - Once everything is moved, delete d

2. Coalesce Leaf with Neighbor



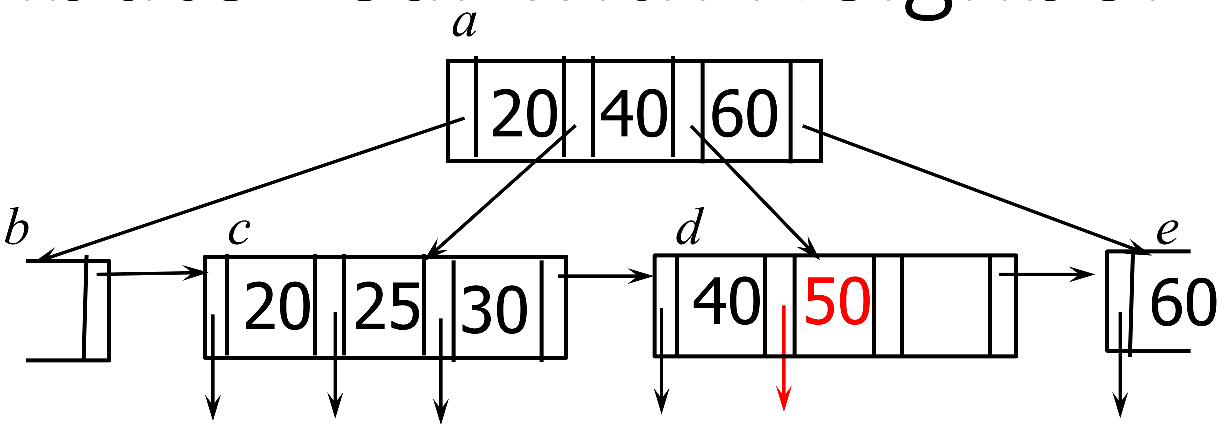
- Delete 50
  - After leaf node merge,
    - From its parent, delete the pointer and key to the deleted node

2. Coalesce Leaf with Neighbor



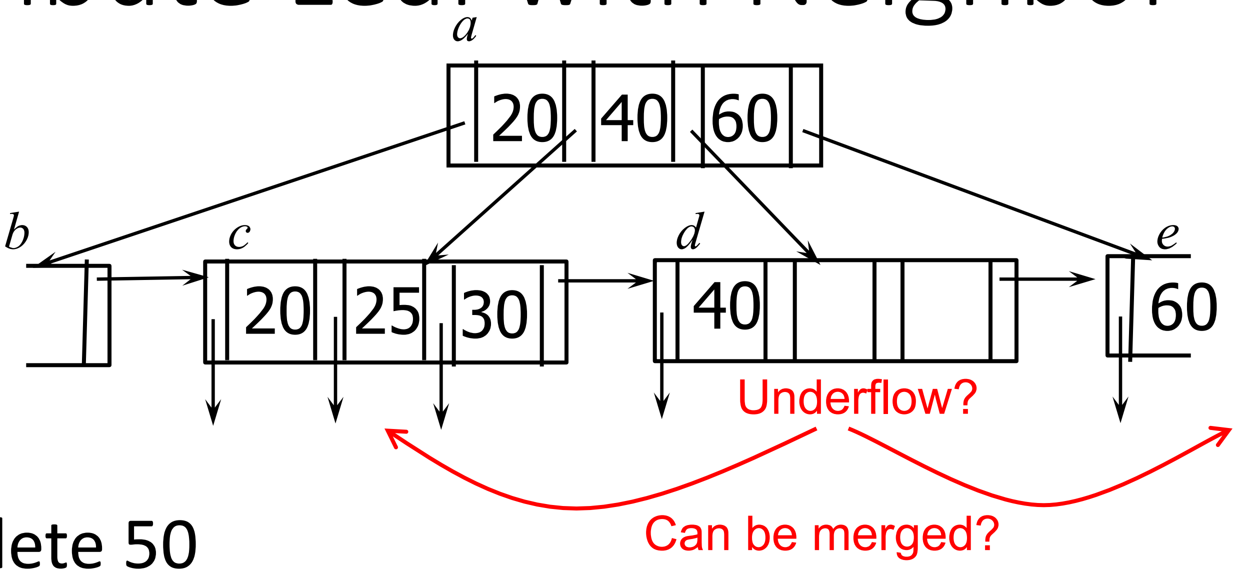
- Delete 50
  - Check underflow at a. Min 2 ptrs, currently 3

3. Redistribute Leaf with Neighbor



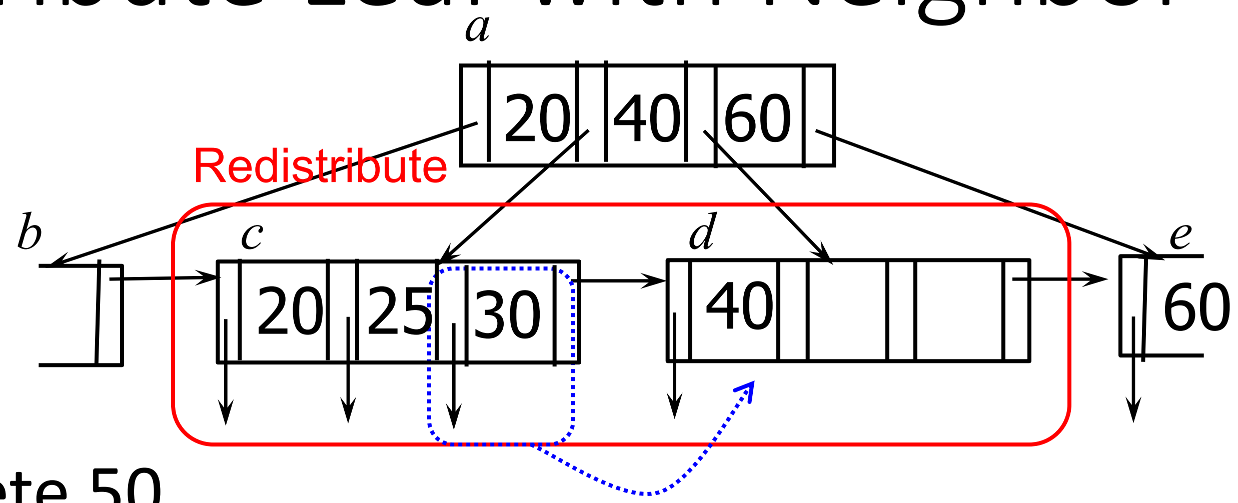
- Delete 50

3. Redistribute Leaf with Neighbor



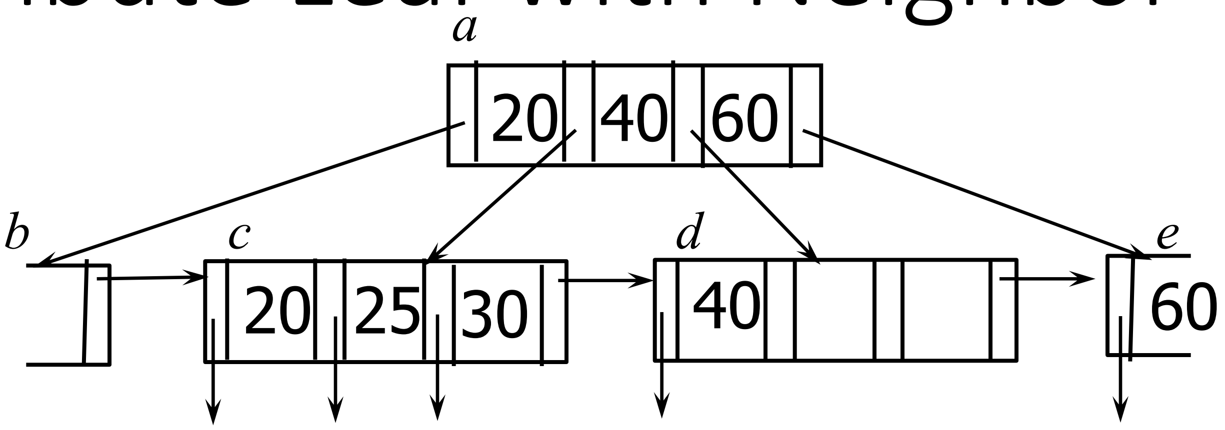
- Delete 50
  - Underflow? Min 3 ptrs, currently 2
  - Check if d can be merged with its sibling c or e
  - If not, redistribute the keys in d with a sibling
    - Say, with c

3. Redistribute Leaf with Neighbor



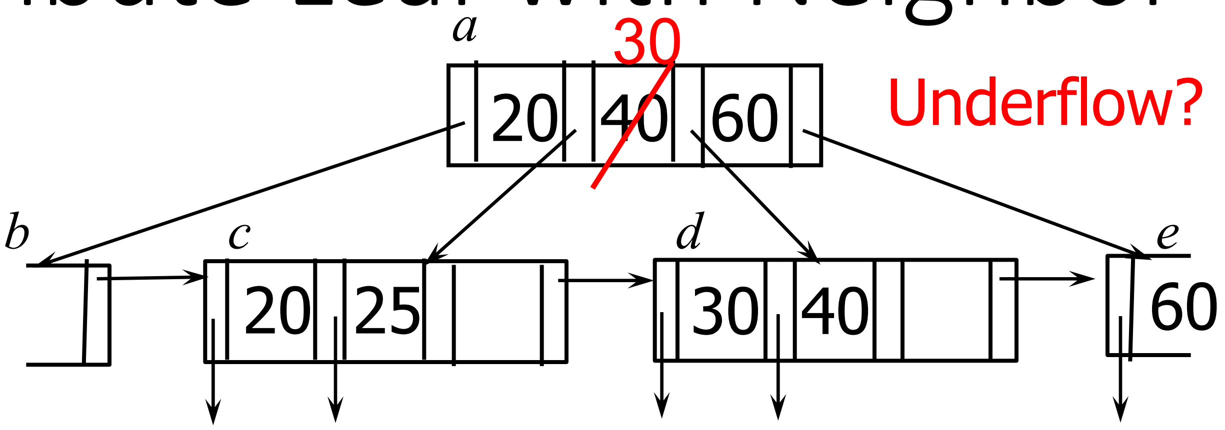
- Delete 50
  - Redistribute c and d, so that nodes c and d are roughly "half full"
    - Move the key 30 and its tuple pointer to the d

3. Redistribute Leaf with Neighbor



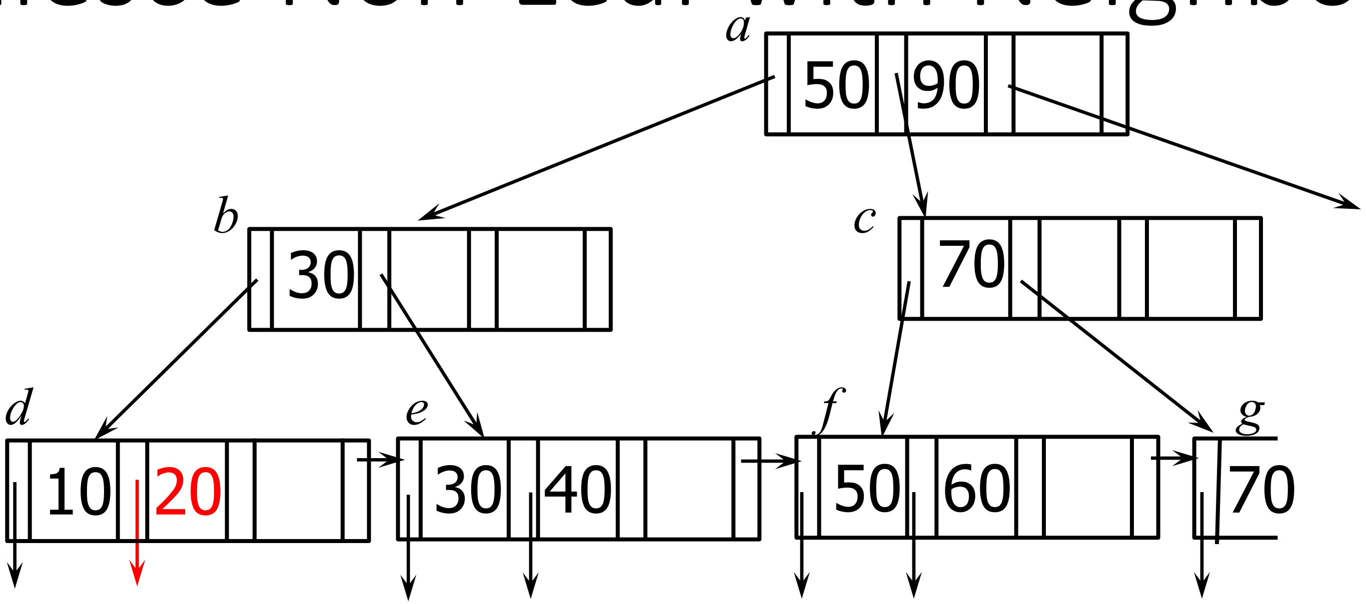
- Delete 50
  - Update the key in the parent

3. Redistribute Leaf with Neighbor



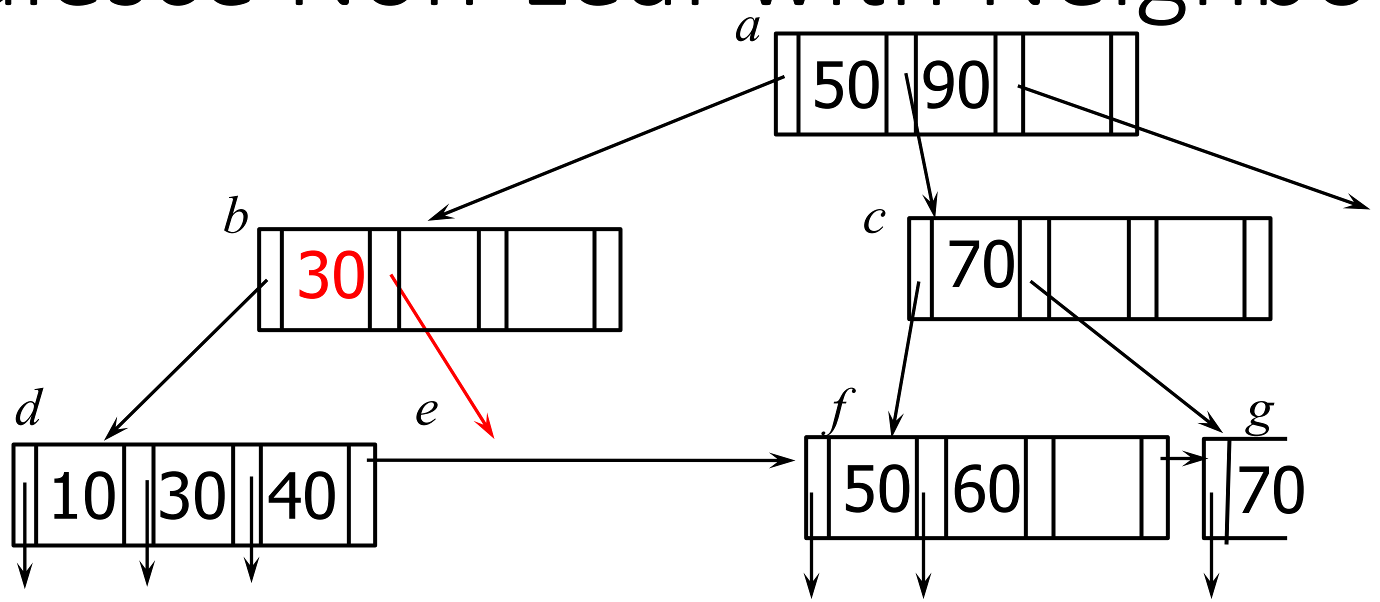
- Delete 50
  - No underflow at a. Done.

4. Coalesce Non-Leaf with Neighbor



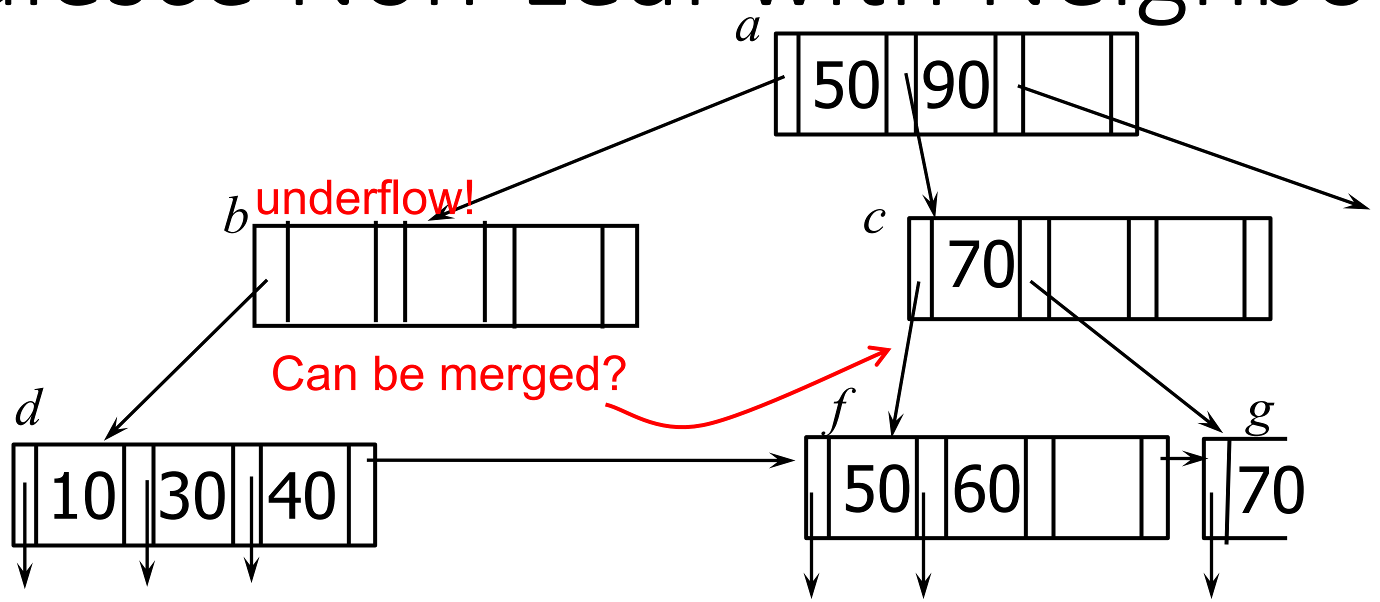
- Delete 20
  - Underflow! Merge d with e.
    - Move everything in the right to the left

4. Coalesce Non-Leaf with Neighbor



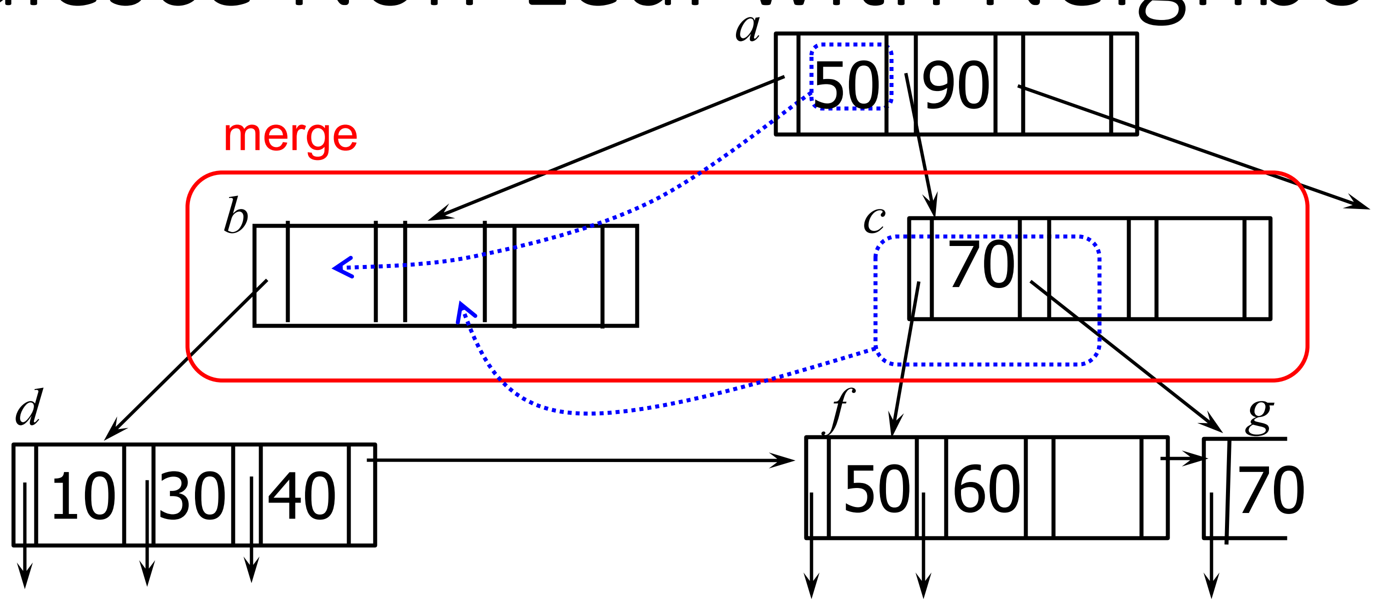
- Delete 20
  - From the parent node, delete pointer and key to the deleted node

4. Coalesce Non-Leaf with Neighbor



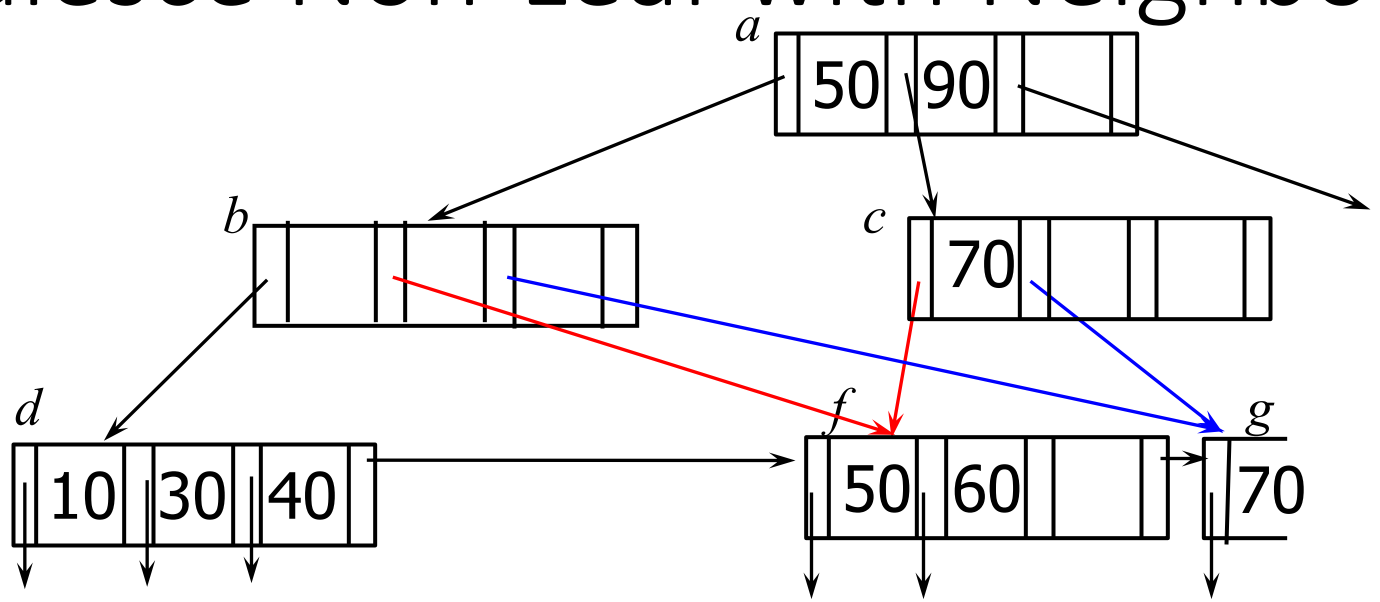
- Delete 20
  - Underflow at b? Min 2 ptrs, currently 1.
  - Try to merge with its sibling.
    - Nodes b and c: 3 ptrs in total. Max 4 ptrs.
    - Merge b and c.

4. Coalesce Non-Leaf with Neighbor



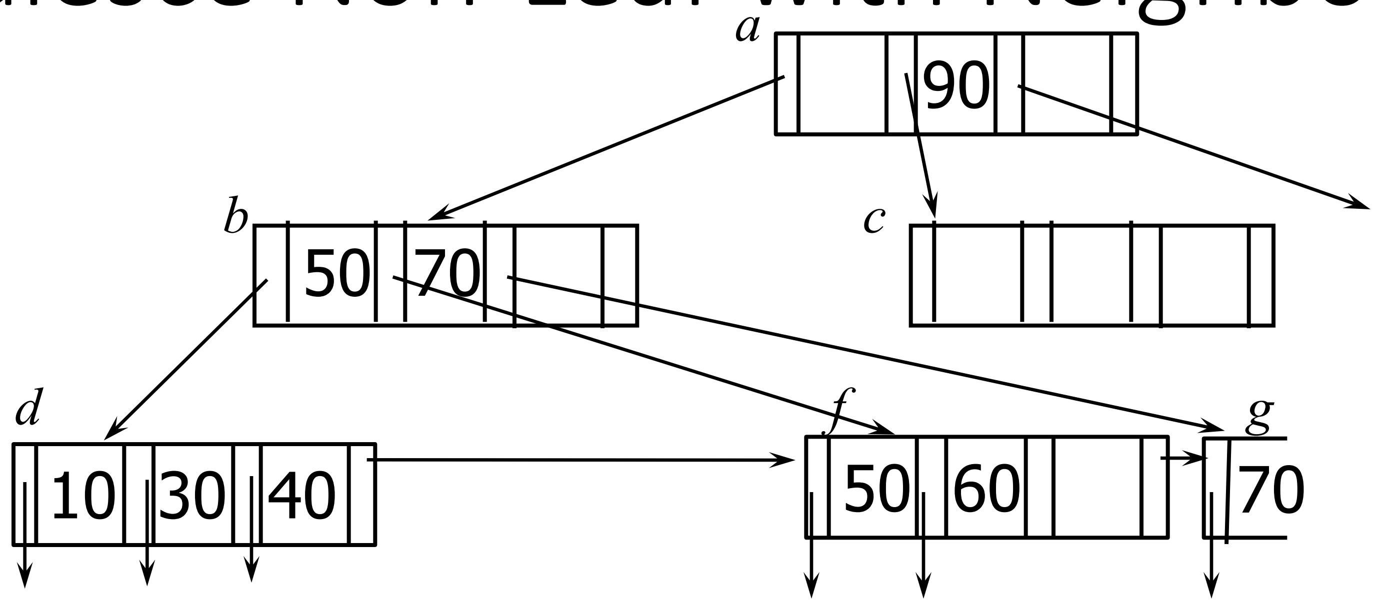
- Delete 20
  - Merge b and c
    - Pull down the mid-key 50 in the parent node
    - Move everything in the right node to the left.
- Very important: when we merge ***non-leaf nodes***, we always pull down the mid-key in the parent and place it in the merged node.

4. Coalesce Non-Leaf with Neighbor



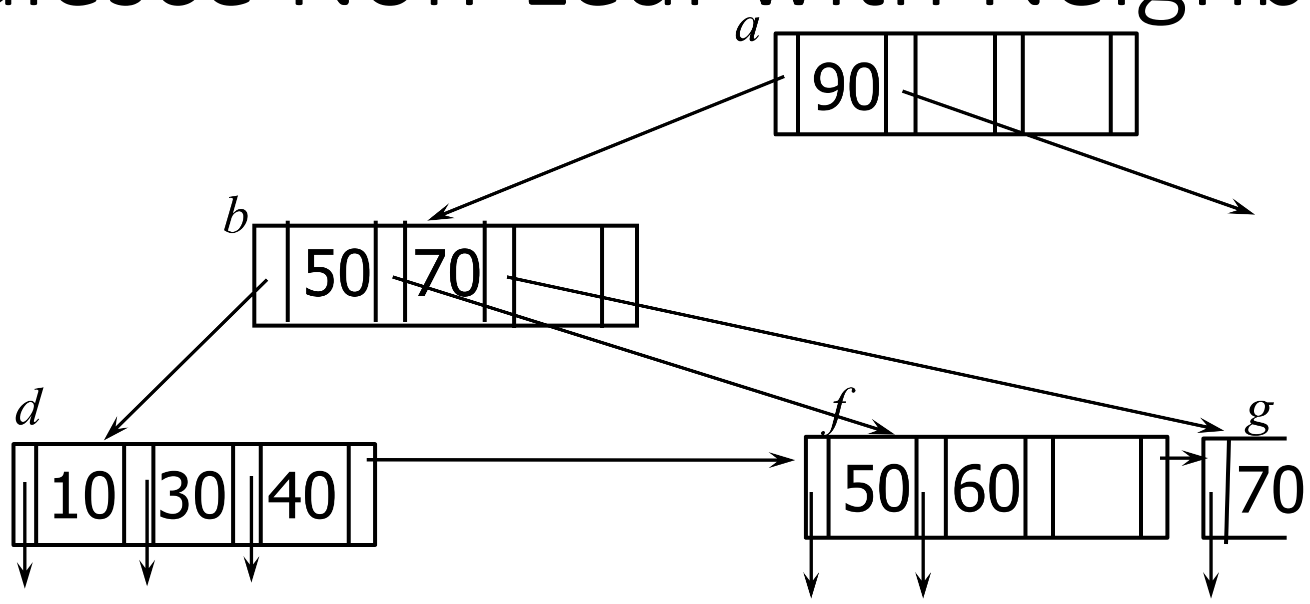
- Delete 20
  - Merge b and c
    - Pull down the mid-key 50 in the parent node
    - Move everything in the right node to the left.
- Very important: when we merge *non-leaf nodes*, we always pull down the mid-key in the parent and place it in the merged node.

4. Coalesce Non-Leaf with Neighbor



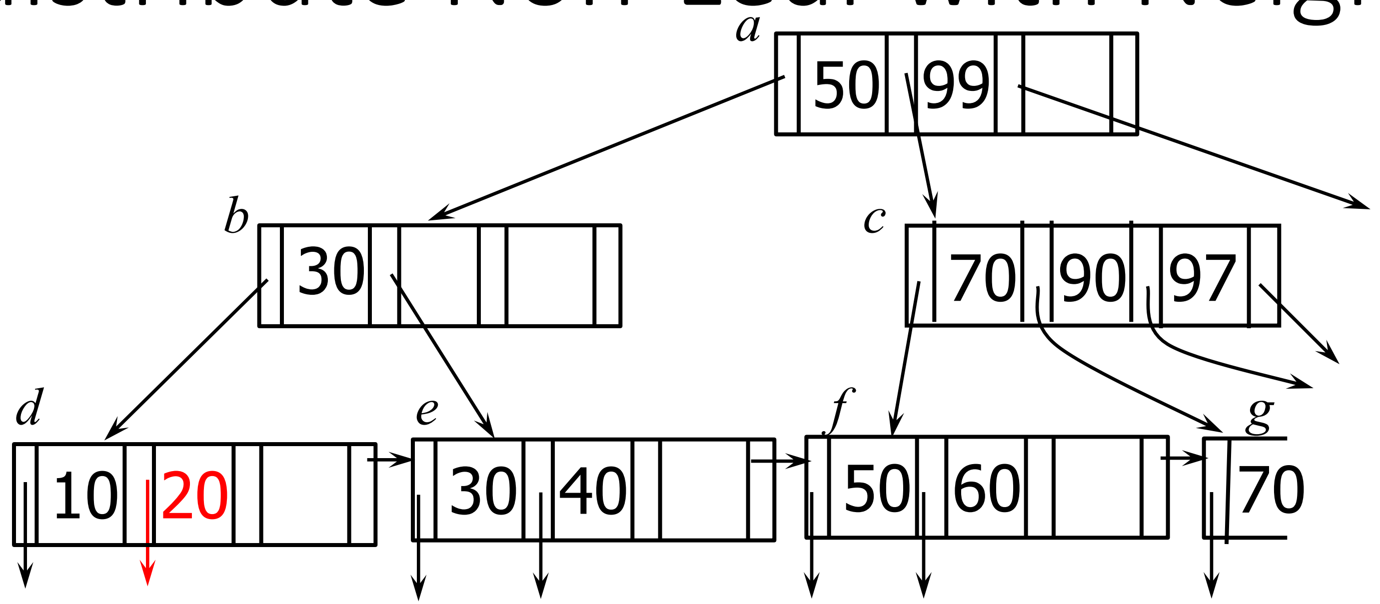
- Delete 20
  - Delete pointer to the merged node.

4. Coalesce Non-Leaf with Neighbor



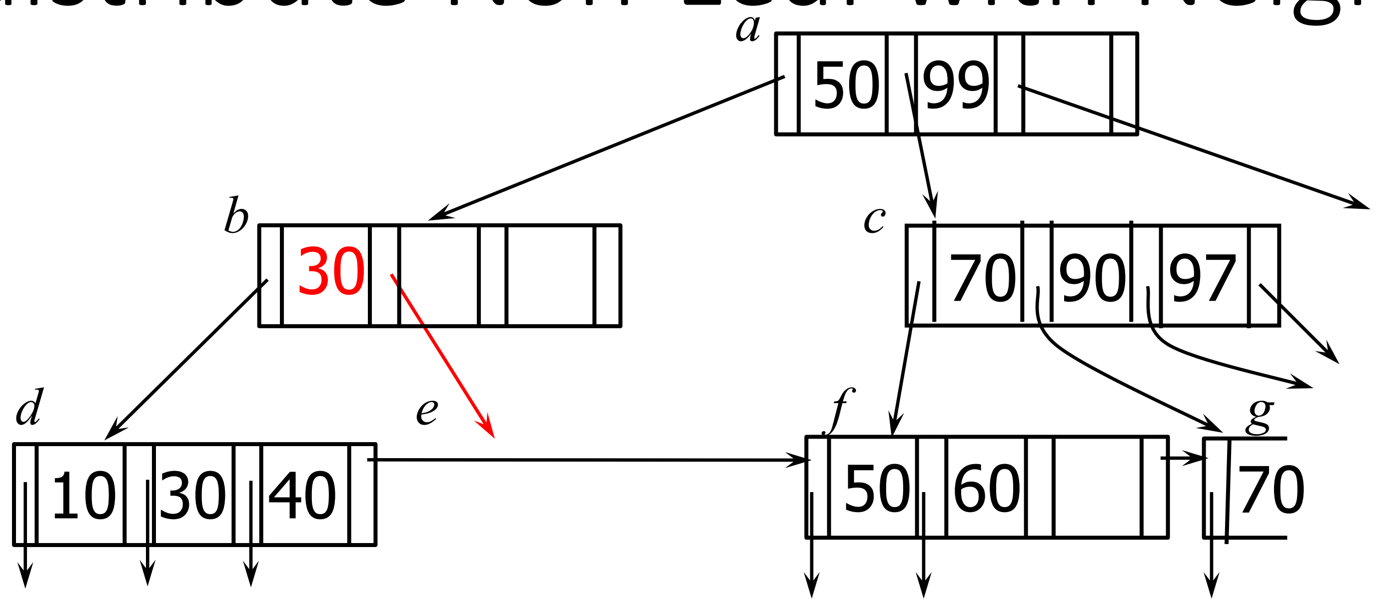
- Delete 20
  - Underflow at *a*? Min 2 ptrs. Currently 2. Done.

5. Redistribute Non-Leaf with Neighbor



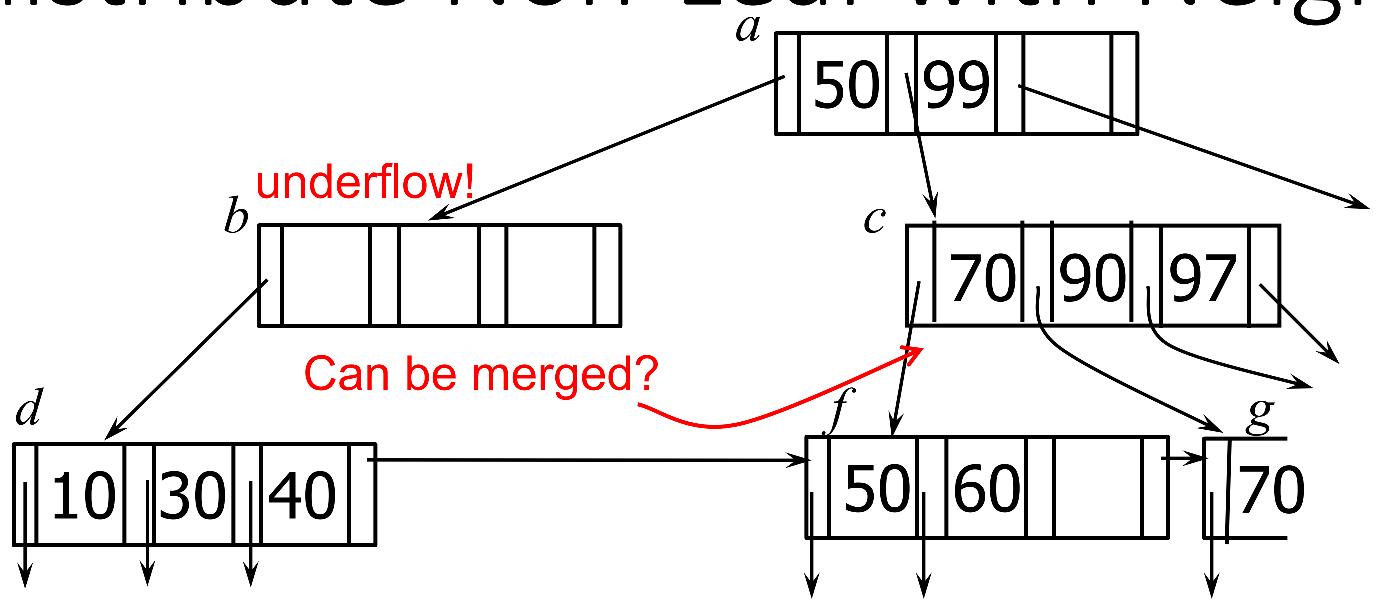
- Delete 20
  - Underflow! Merge d with e.

5. Redistribute Non-Leaf with Neighbor



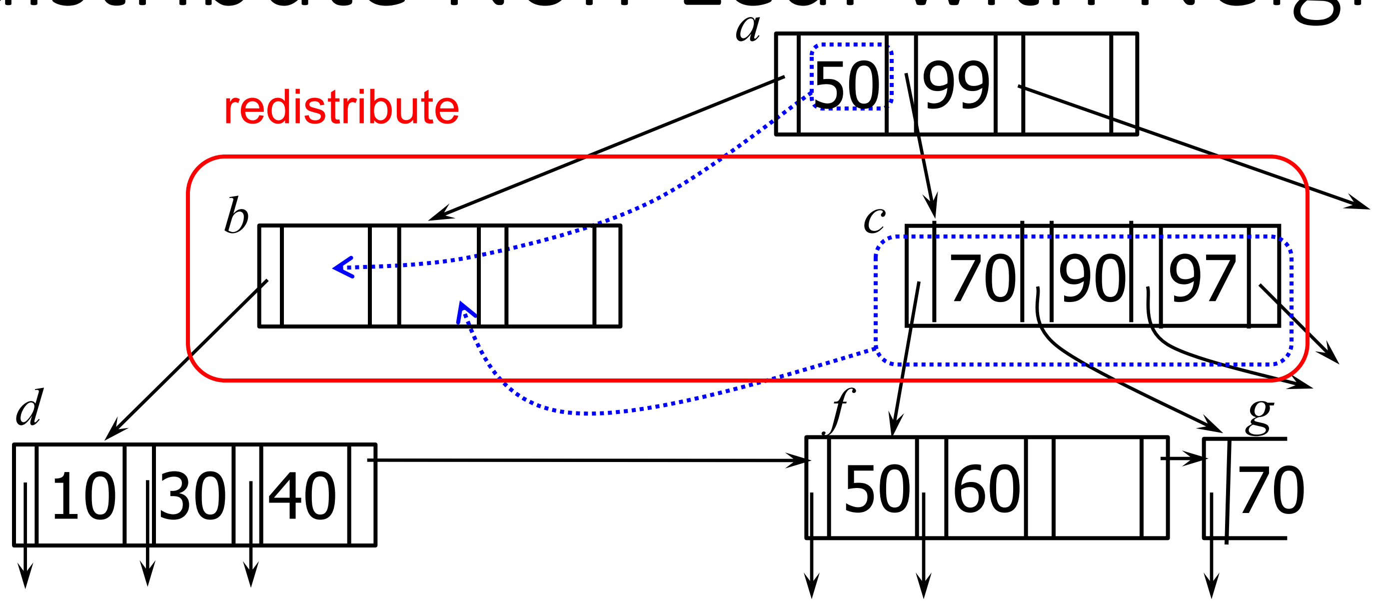
- Delete 20
  - After merge, remove the key and ptr to the deleted node from the parent

5. Redistribute Non-Leaf with Neighbor



- Delete 20
  - Underflow at b? Min 2 ptrs, currently 1.
  - Merge b with c? Max 4 ptrs, 5 ptrs in total.
  - If cannot be merged, redistribute the keys with a sibling.
    - Redistribute b and c

5. Redistribute Non-Leaf with Neighbor

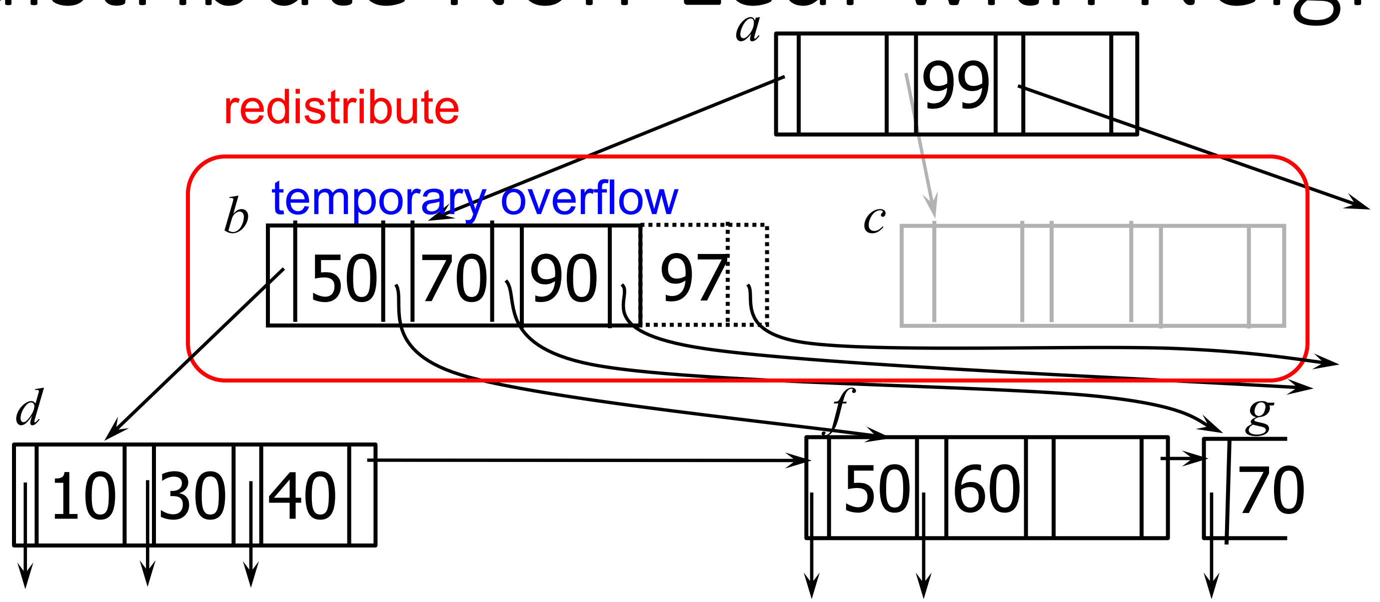


- Delete 20

<u>Redistribution at a non-leaf node is done in two steps</u>.

Step 1: Temporarily, make the left node b “overflow” by pulling down the mid-key and moving everything to the left.

5. Redistribute Non-Leaf with Neighbor

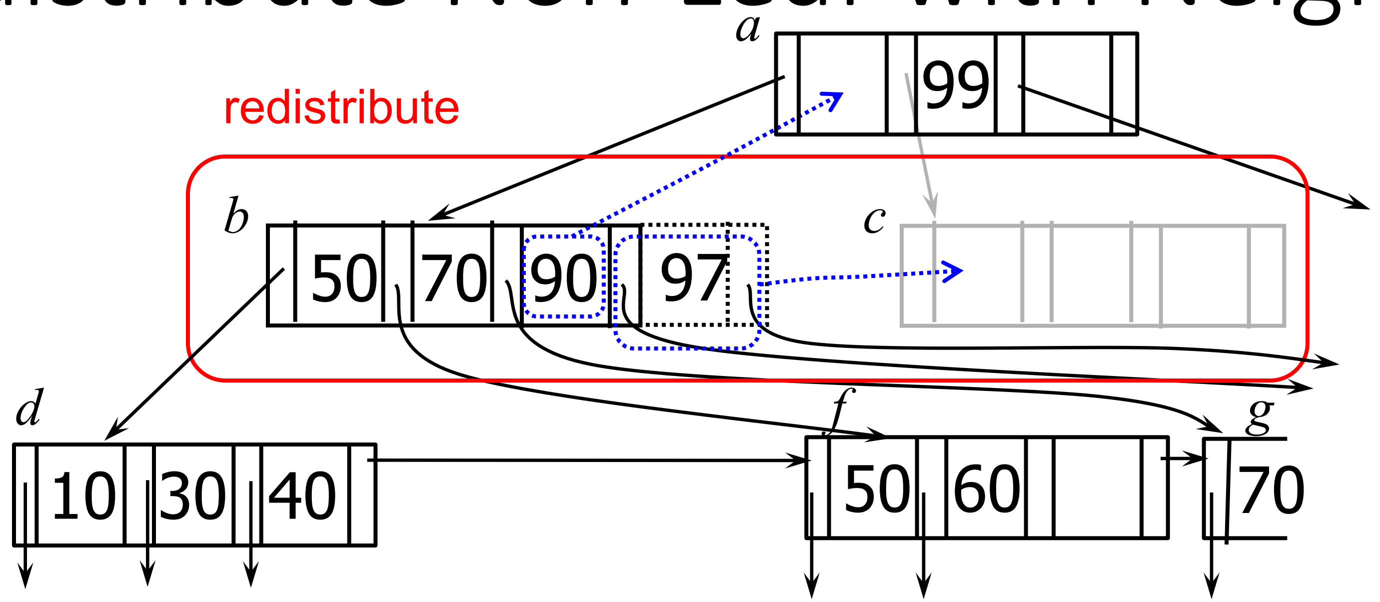


- Delete 20

Step 2: Apply the "overflow handling algorithm" (the same algorithm used for B+tree insertion) to the overflowed node

  - Detailed algorithm in the next slide

5. Redistribute Non-Leaf with Neighbor

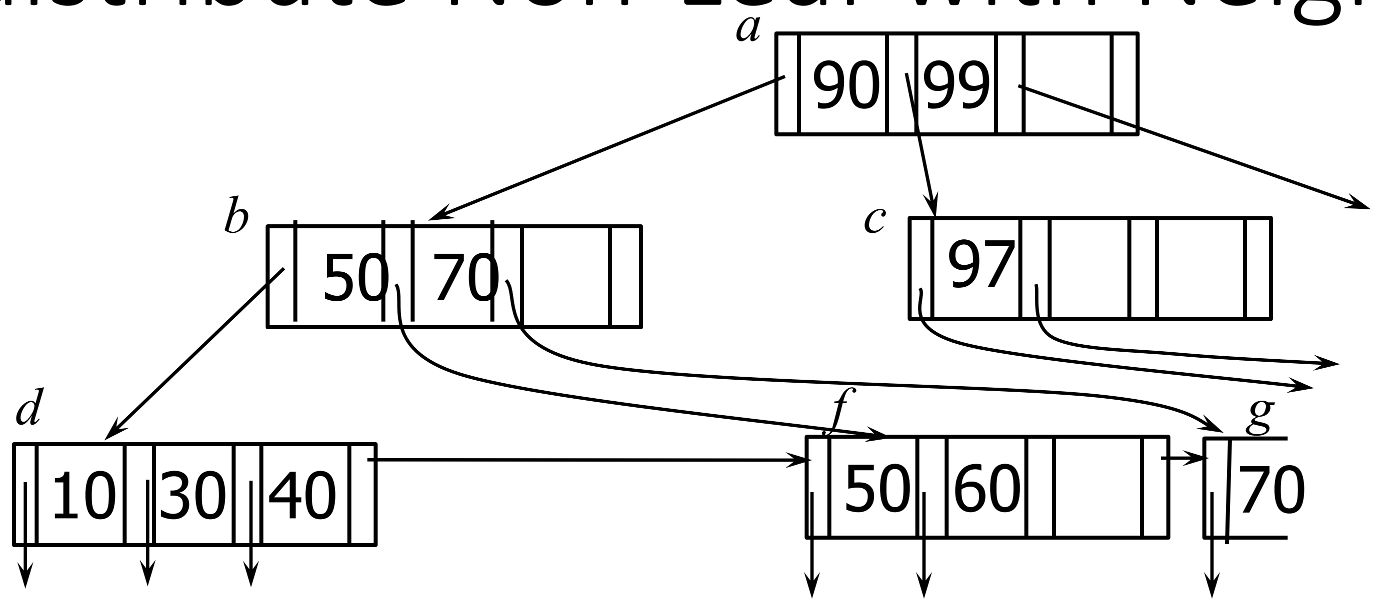


- Delete 20

Step 2: "overflow handling algorithm"

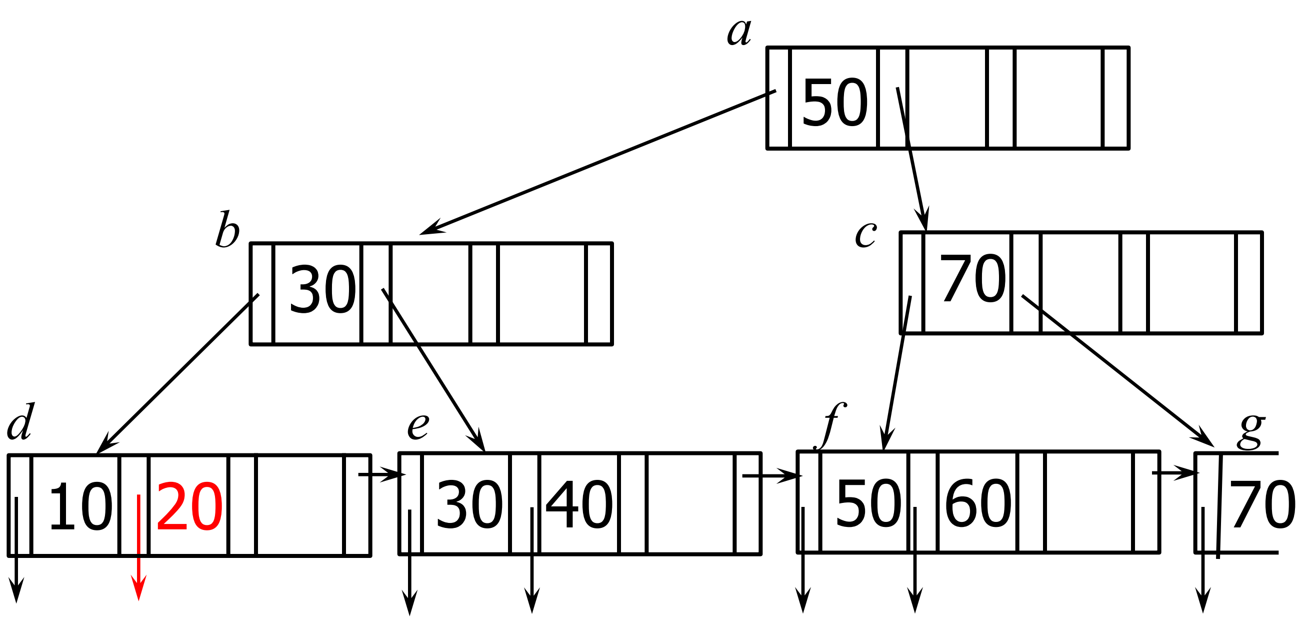
- Pick the mid-key (say 90) in the node and move it to parent.
- Move everything to the right of 90 to the empty node c.

5. Redistribute Non-Leaf with Neighbor



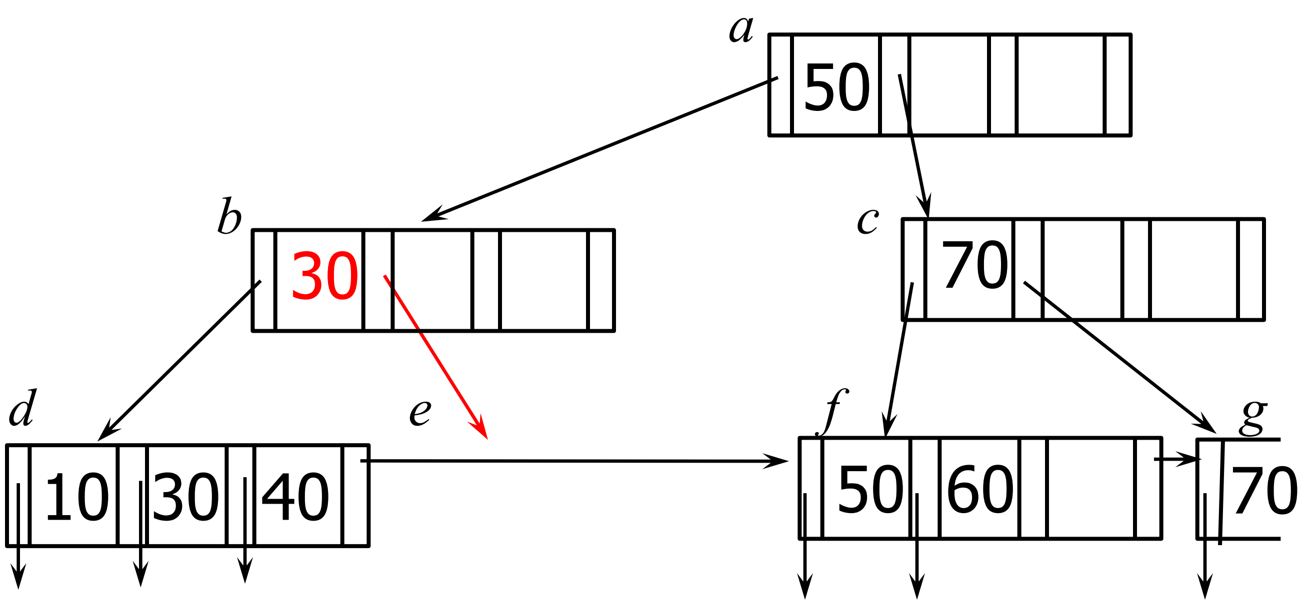
- Delete 20
  - Underflow at a? Min 2 ptrs, currently 3. Done

6. Reduce Tree Depth



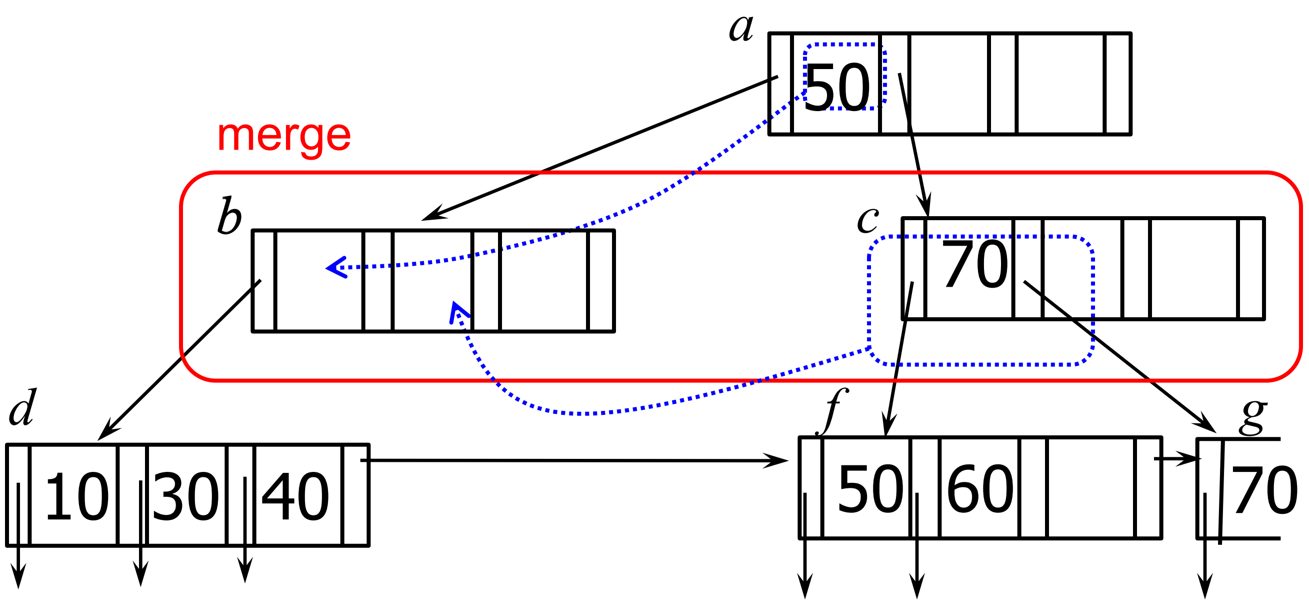
- Delete 20
  - Underflow! Merge d with e.
  - Move everything in the right node to the left

6. Reduce Tree Depth



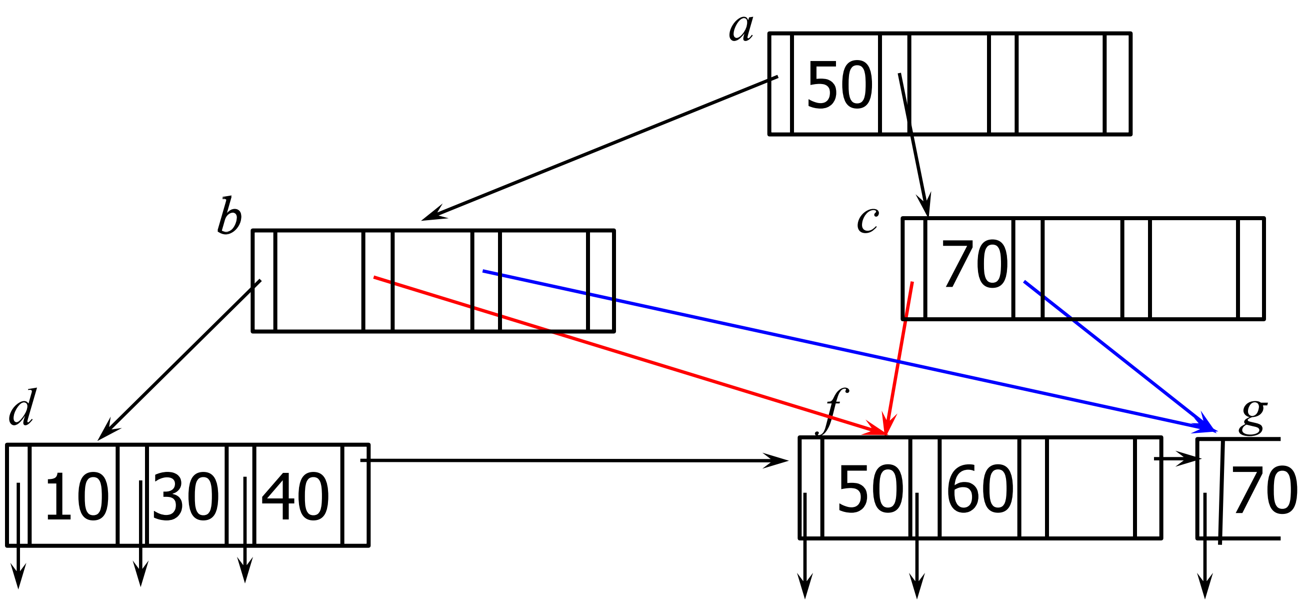
- Delete 20
    - From the parent node, delete pointer and key to the deleted node

6. Reduce Tree Depth



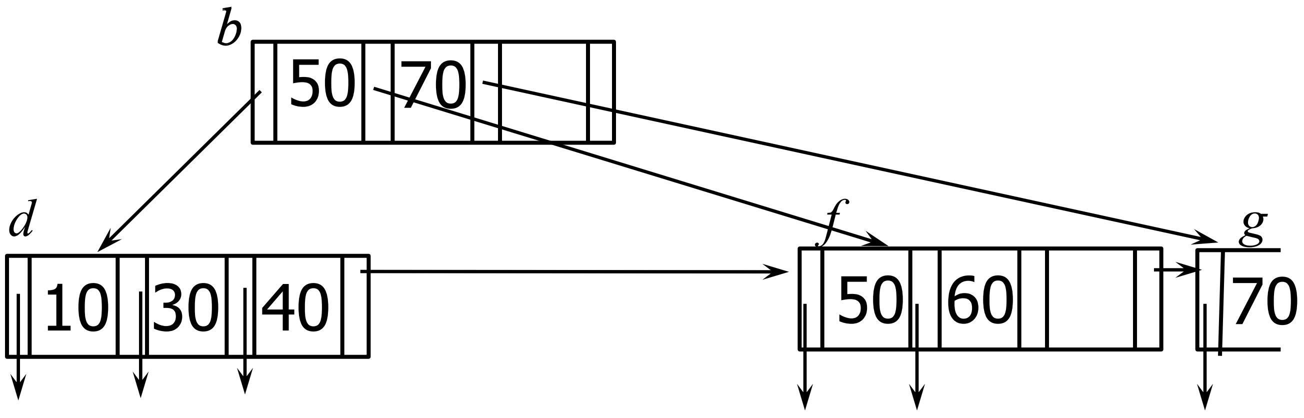
- Delete 20
  - Merge b and c
    - Pull down the mid-key 50 in the parent node
    - Move everything in the right node to the left.

6. Reduce Tree Depth



- Delete 20
  - After merging b and c, remove empty root node
  - Tree depth is decreased by one

6. Reduce Tree Depth



- Delete 20

Important Points

- Remember:
  - For *leaf node* merging, we *delete* the mid-key from the parent
  - For *non-leaf node* merging/redistribution, we *pull down* the mid-key from their parent.
- Exact algorithm: Figure 14.21

Where does *n* come from?

- *n* determined by
  - Size of a node
  - Size of search key
  - Size of an index pointer
- Q: 1024B node, 10B key, 8B ptr → *n*?

$$8n + 10(n-1) \leq 1024$$

$$8n + 10n - 10 \leq 1024$$

$$18n \leq 1024 + 10 = 1034$$

$$n \leq \frac{1034}{18} = 57.44$$

Range Search on B+tree

- SELECT *
  FROM Student
  WHERE sid > 60?